## What is a function?

## Quick Check

Name and give a description of a "machine" that takes an input and produces an output?

## Function Machine



$$
f(x)=x^{2}+1
$$

## Reading a Function Rule

$$
f(x)=2 x^{2}-4 x+1 \quad \text { view as } \rightarrow \quad f(\quad)=2(\quad)^{2}-4(\quad)+1
$$

a. $f(-2)=$
b. $f(2 a)=$
c. $f\left(\frac{x}{2}\right)=$
d. $f\left(x^{2}-1\right)-f(2)=$

Evaluate the function at the given input. [ $a, b, h$ are real numbers.]

1. For the function $f$ defined by $f(x)=x^{2}+7$, evaluation each expression:
a. $f(3 a)$
b. $f(b-1)$
c. $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$
2. If $f(x)=2 x^{2}+5 x+1$ and $h \neq 0$, evaluate $\frac{f(a+h)-f(a)}{h}$

## Finding the domain $\rightarrow$

$$
f(x)=\frac{1}{x-4}
$$

$$
g(x)=\sqrt{x}
$$

$$
h(x)=\frac{\sqrt{x}}{x-4}
$$

## Find the domain of each function.

$$
f(x)=\frac{5}{x^{2}-x}
$$

$$
g(x)=\sqrt{9-x^{2}}
$$

$$
h(t)=\frac{t}{\sqrt{t+1}}
$$

What does it mean for a function to be defined or undefined?

## Mapping Function definition $\longleftrightarrow$ graph

If $f$ is a function with domain $A$, then the graph of $f$ is the set of (input, output) pairs

$$
\{(x, f(x)) \mid x \in A\}
$$

The graph of the function $f$ is the graph of the equation $y=f(x)$. The outputs are assigned to the $y$-axis.


## Piece-wise defined functions

Functions defined by different formulas in different parts of their domain.


$$
f(x)= \begin{cases}-x & \text { if } x<0 \\ x^{2} & \text { if } 0 \leq x \leq 1 \\ 1 & \text { if } x>1\end{cases}
$$

What situations in life can be modeled with these functions?

## Graph $\longleftrightarrow$ Equation

1. 



Using the graph, write the definition for the function.
2. $\quad f(x)= \begin{cases}x+3 & \text { if }-2 \leq x<1 \\ 5 & \text { if } x=1 \\ -x+2 & \text { if } x>1\end{cases}$ Graph the piecewise-defined function.

## Evaluate Piece-wise Functions

1. A function $f$ is defined by

$$
f(x)= \begin{cases}1-x & \text { if } x \leq-1 \\ x^{2} & \text { if } x>-1\end{cases}
$$

Evaluate $f(-5), f(-1), f(0), f(100)$, and sketch the graph.
2. Given a piecewise defined function:

$$
g(t)= \begin{cases}t^{2}+2 & \text { if } t \leq 2 \\ t^{2} & \text { if } t>2\end{cases}
$$

Find $g(-2), g(0), g\left(s^{2}+2\right)$ where $s$ is a real number.

How does the Vertical Line Test help us inspect if a graph shows a function?



## Are these the equations of functions?

1. $y^{2}=x^{2}-4$
2. $\left(x^{2}+y^{2}\right)^{2}=100\left(x^{2}-y^{2}\right)$
3. $x^{2}+y=4$
4. $\left(x^{2}+4\right) y=8$
5. $x^{2 / 3}+y^{2 / 3}=5$
6. $x y=1$

Use Desmos.com or your calculator as necessary. Develop a graphical or algebraic reasoning to explain your answer.

## Symmetry

A function is called an even function if

$$
f(-x)=f(x)
$$



A function is called an odd function if

$$
f(-x)=-f(x)
$$



Why are we not talking about symmetry about the $x$-axis for functions?

Determine whether $f$ is even, odd, or neither algebraically. Use a graphing calculator to verify your result.

1. $f(x)=\frac{x}{x^{2}+1}$
2. $g(x)=1+3 x^{3}-x^{5}$
3. $h(x)=x|x|$
4. $p(t)=\frac{t^{2}}{t^{4}+1}$
