

What approaches can we use to estimate limits?

Quick Check

Look at the graph of $f(x) = \frac{x^2 - 1}{x - 1}$

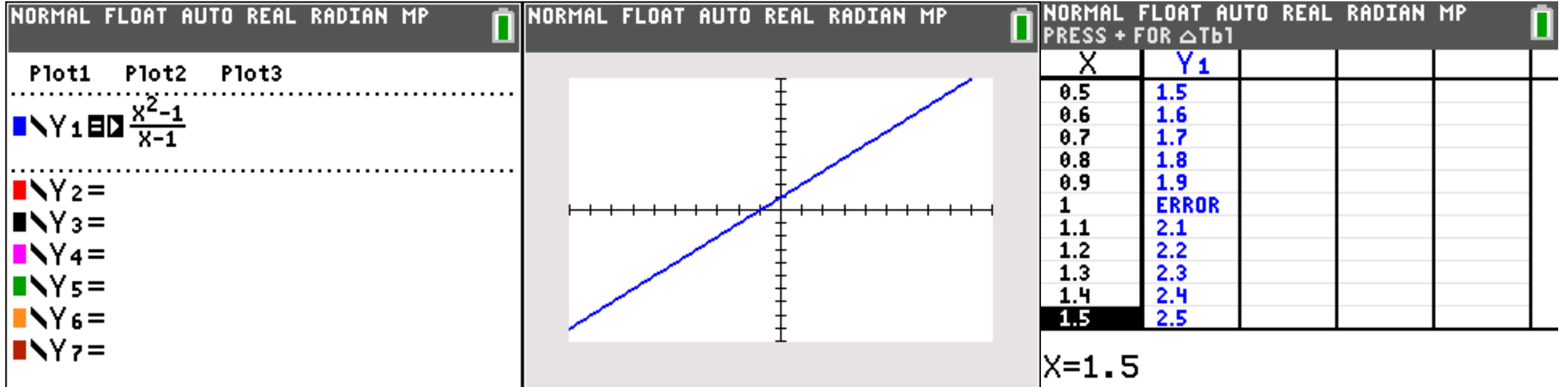
What happens when $x = 1$?

What happens when x is near 1?

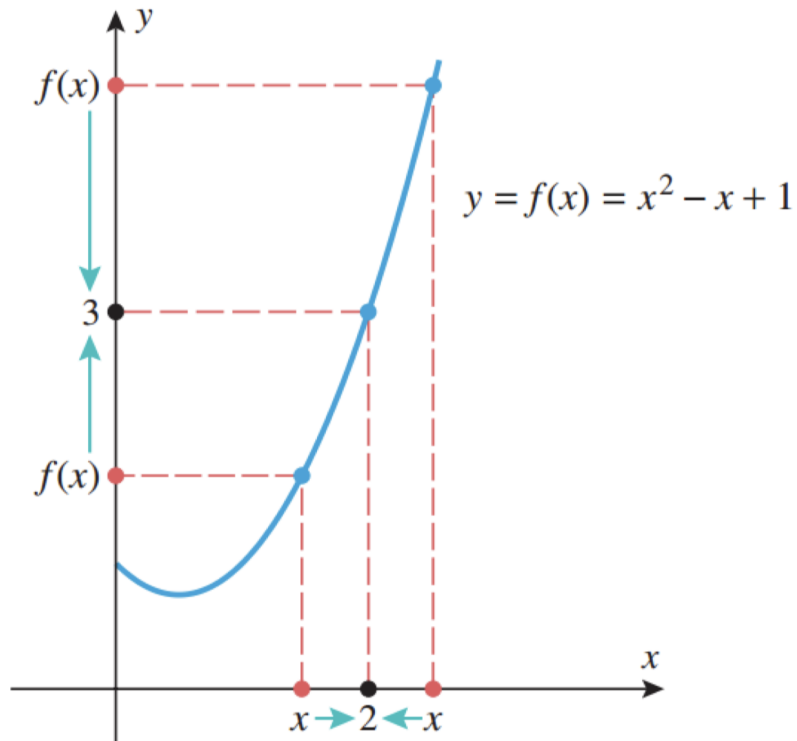
What happens to $f(x)$ when x is near 1?

In other words,

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$



Graphical, Tabular, and Algebraic View



$$\lim_{x \rightarrow 2} (x^2 - x + 1) = 3$$

The limit of $f(x)$ as x approaches 2 from both sides is 3.

x	1.0	1.5	1.9	1.95	1.99	1.995	1.999	2	2.001	2.005	2.01	2.05	2.1	2.5	3.0
$f(x)$	1.000000	1.750000	2.710000	2.852500	2.970100	2.985025	2.997001		3.003001	3.015025	3.030100	3.152500	3.310000	4.750000	7.000000

Left side

Right side

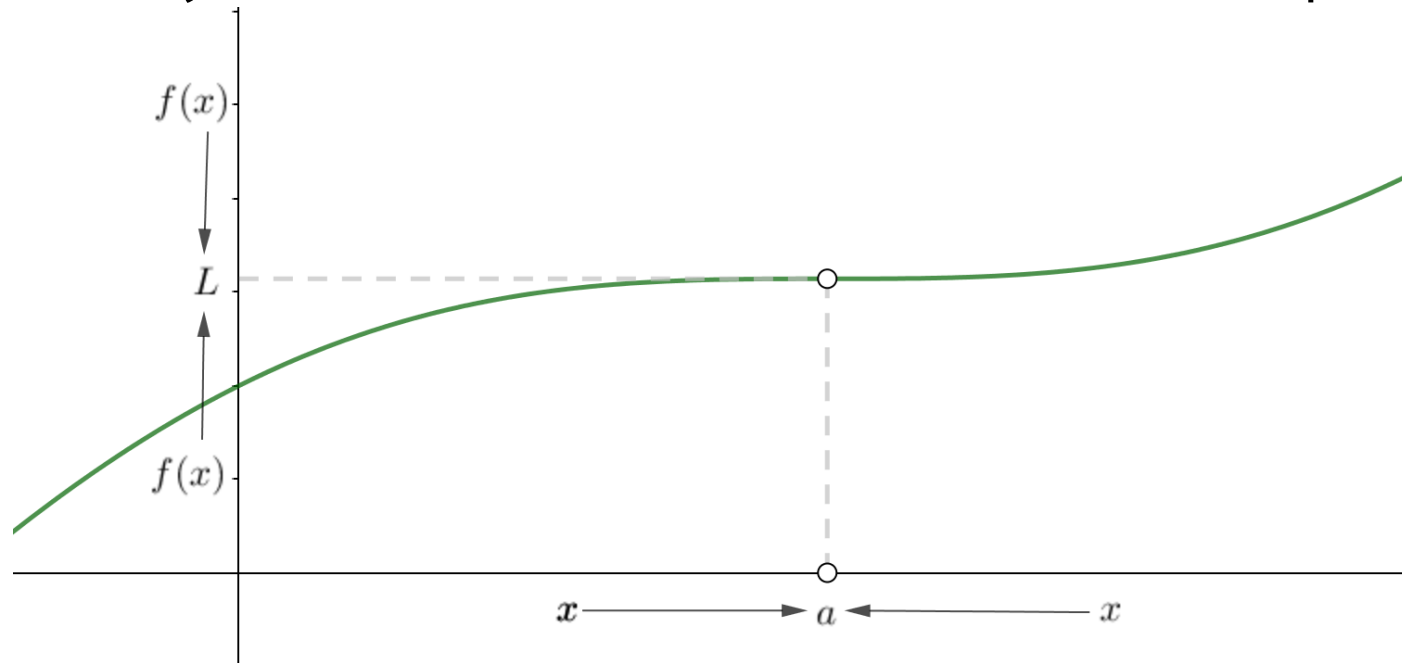
Definition

We write

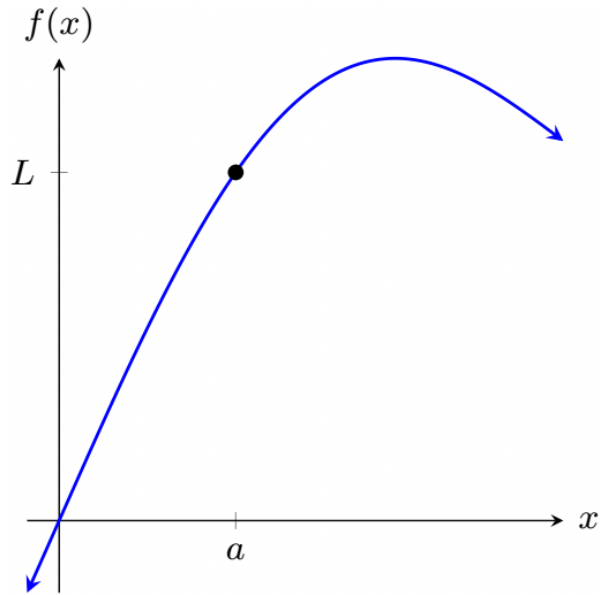
$$\lim_{x \rightarrow a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a , equals L "

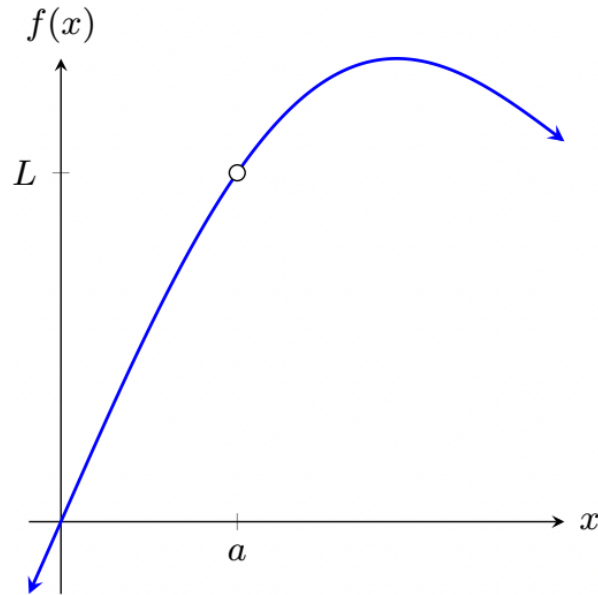
if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a .



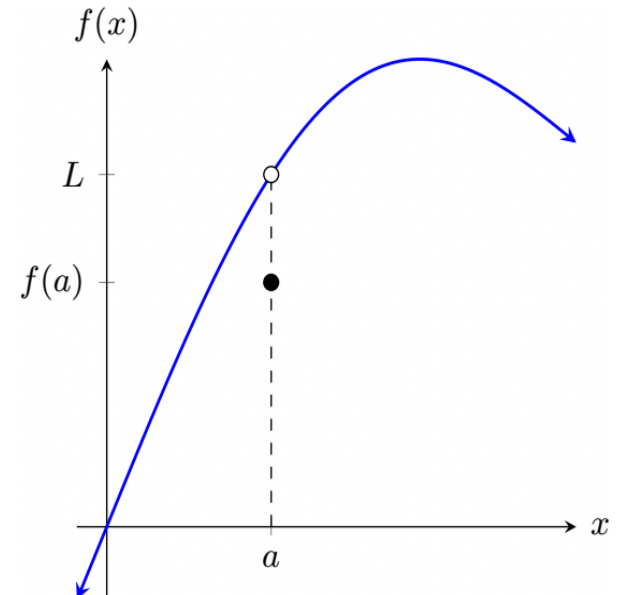
What is the limit in each case?



$$\lim_{x \rightarrow a} f(x) =$$

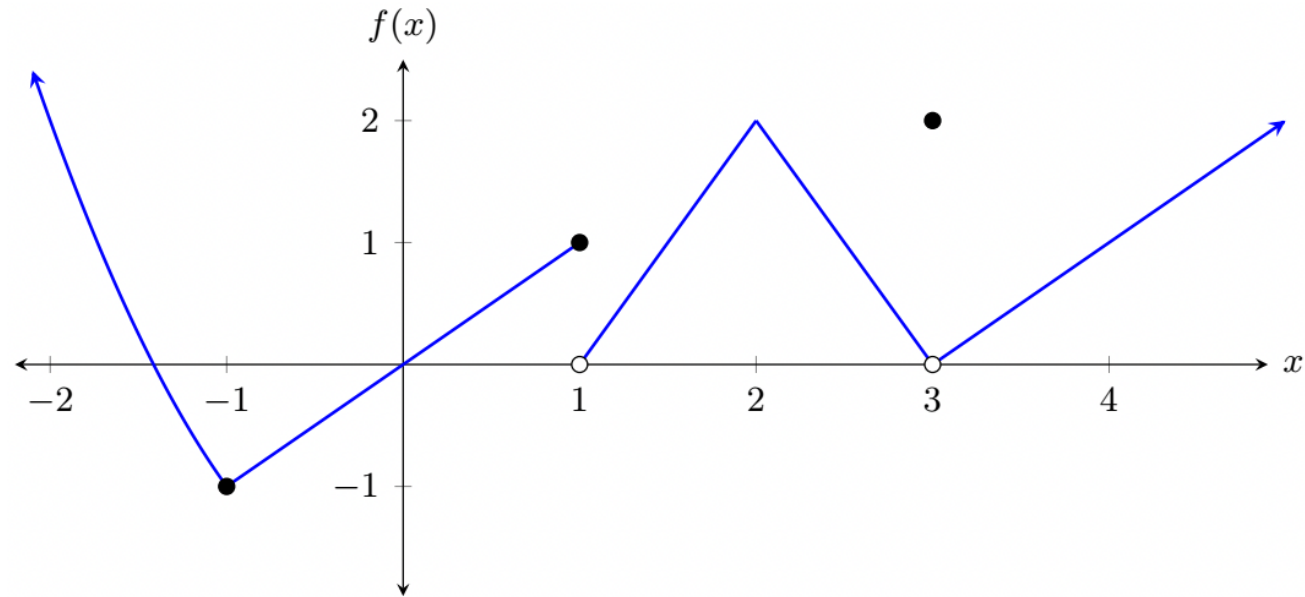


$$\lim_{x \rightarrow a} f(x) =$$



$$\lim_{x \rightarrow a} f(x) =$$

Use the graph to find each limit, if it exists.



1. $\lim_{x \rightarrow -1} f(x)$

2. $\lim_{x \rightarrow 1} f(x)$

3. $\lim_{x \rightarrow 2} f(x)$

4. $\lim_{x \rightarrow 3} f(x)$

5. $\lim_{x \rightarrow 4} f(x)$

In what ways can a limit fail to exist?

$$1. \lim_{x \rightarrow 0} \frac{|x|}{x}$$

$$2. \lim_{x \rightarrow 1} \frac{1}{(x - 1)^2}$$

$$3. \lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$$

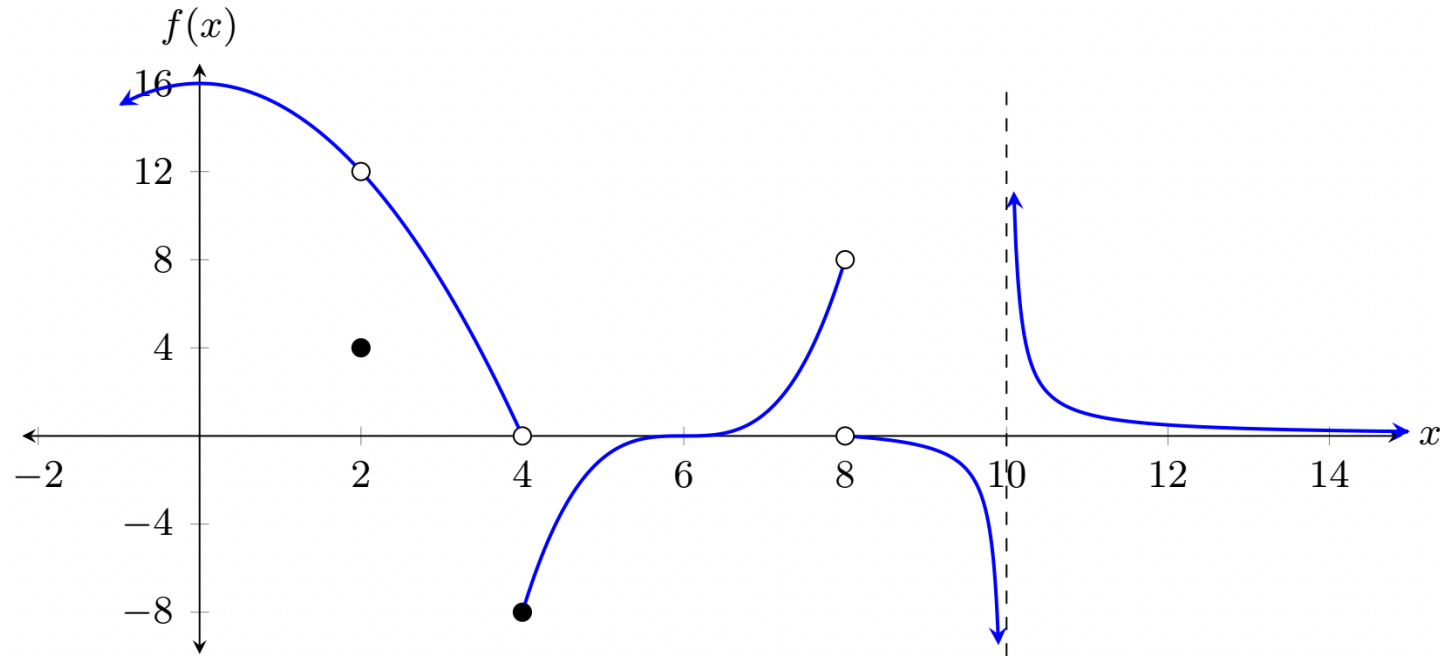
Summary of common behaviors of a function associated with nonexistence of a limit

$$\lim_{x \rightarrow a} f(x)$$

Does Not Exist because...

1. $f(x)$ approaches a different number from the right side of a than it approaches from the left side of a .
2. $f(x)$ increases or decreases without bound as x approaches a .
3. $f(x)$ oscillates between two fixed values as x approaches a .

Find the following limits or explain why they don't exist.



1. $f(2)$

$$\lim_{x \rightarrow 2} f(x)$$

2. $f(4)$

$$\lim_{x \rightarrow 4} f(x)$$

3. $f(6)$

$$\lim_{x \rightarrow 6} f(x)$$

4. $f(8)$

$$\lim_{x \rightarrow 8} f(x)$$

5. $f(10)$

$$\lim_{x \rightarrow 10} f(x)$$

A question of equality

1. What is wrong with the following equation?

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

2. Keeping part (1) in mind, explain why the following is correct.

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} (x + 3)$$