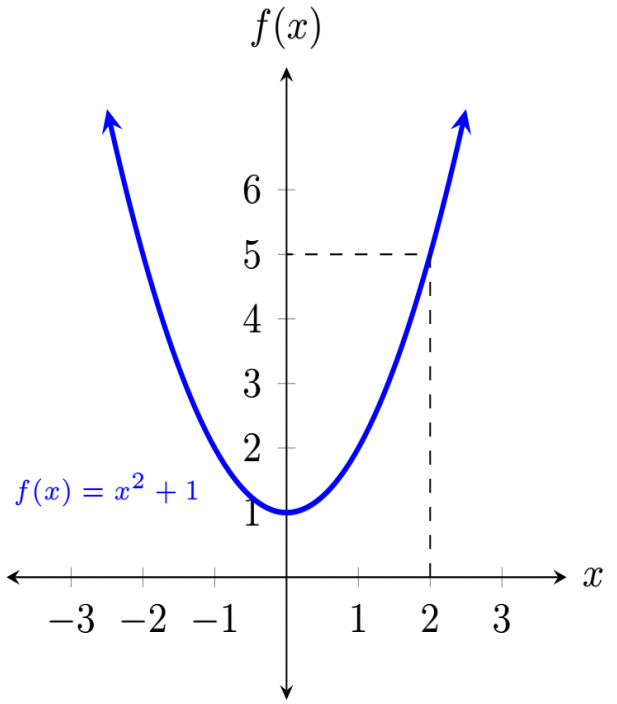
How do we calculate limits using limit laws?

Quick Check

Consider the following function's value and its limit at $x = 2$.

$$\lim_{x \to 2}(x^2 + 1) = \qquad\qquad f(2) =$$

We know that the limit of the function as x approaches a does not depend on the value of the function at $x = a$. However, it may happen, that the limit is $f(a)$. **Describe when this could happen or not happen. Sketch graphs to illustrate.**



Basic Limits

Let b and a be real numbers and let n be a positive integer.

1. $\lim_{x \to a} b = b$
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} x^n = a^n$

🤔 Draw a separate graph for each limit. Explain the reasoning.

Limit of a function involving a radical

Let n be a positive integer. The following limit is valid for all a if n is odd, and is valid for $a > 0$ if n is even.

$$\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$$

🤔 What is the meaning of the condition on n, the index of the root?

Find $\lim_{x \to 4} \sqrt[3]{x + 4}$

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

Find the limits.

$$\lim_{x \to 1} (x^2 - 3x + 5)$$

$$\lim_{x \to 1} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$$

$$\lim_{x \to -2} \frac{x + 2}{x^2 - 4}$$

Limit Laws

Let c be a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then

1. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} f(x)$

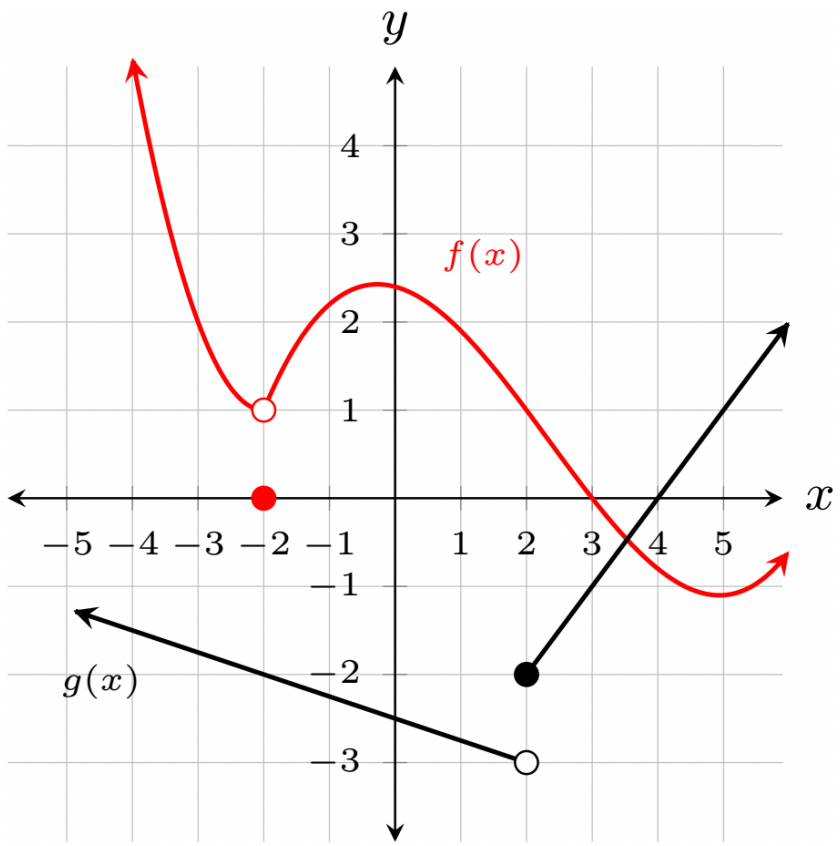
3. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

4. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

🤔 How can I know these are true?

Applying Limit Laws

Use the graphs of f and g to evaluate the following limits, if they exist.



1. $\lim_{x \to -2} [f(x) + 2g(x)]$

2. $\lim_{x \to 2} f(x) \cdot g(x)$

3. $\lim_{x \to 4} \dfrac{f(x)}{g(x)}$

🤔 What are the requirements for applying **limit laws**?

Applying Limit Laws

Given that

$$\lim_{x\to 2} f(x) = 8\,, \quad \lim_{x\to 2} g(x) = -2\,, \quad \lim_{x\to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

a. $\lim_{x\to 2}[f(x) + 4g(x)]$

b. $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$

c. $\lim_{x\to 2}[g(x)]^3$

d. $\lim_{x\to 2} \dfrac{g(x)h(x)}{f(x)}$

Limit of a composite function

If f and g are functions such that $\lim_{x \to a} g(x) = L \quad \text{and} \lim_{x \to L} f(x) = f(L), \text{then}$

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x) \right) = f(L)$$

1. Given

$f(x) = 5 - x$
$g(x) = x^3$

Find $\lim_{x \to 2} g\big(f(x)\big)$

2. Given

$f(x) = 2x^2 - 3x + 1$
$g(x) = \sqrt[3]{x + 6}$

Find $\lim_{x \to 4} g\big(f(x)\big)$

True or False. Explain.

1. If $\lim_{x \to a} f(x) = L$, then $f(a) = L$.

2. If $f(a) = L$, then $\lim_{x \to a} f(x) = L$.

3. If f is undefined at $x = a$, then the limit of $f(x)$ as x approaches a does not exist.

🤔 What proof is needed to show that a statement is true? false?