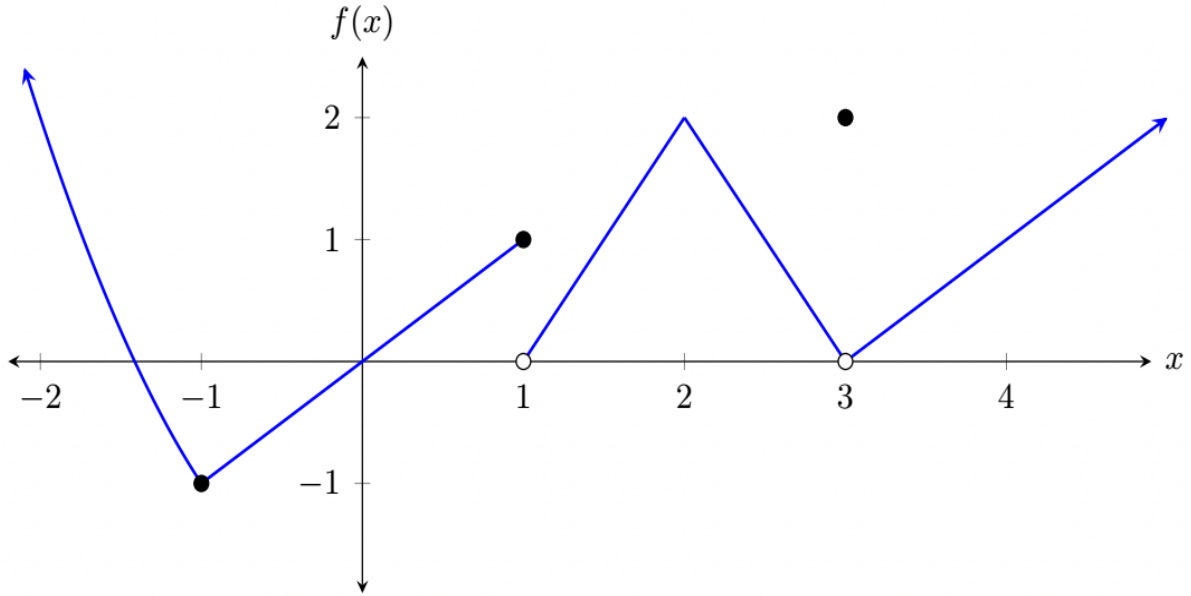
What does it mean to say that a function is continuous?

Quick Check



Find

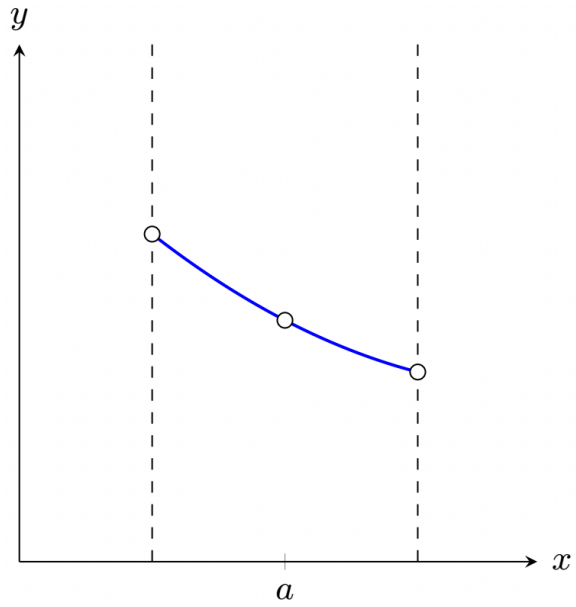
a. $f(a)$

b. $\lim_{x \to a} f(x)$

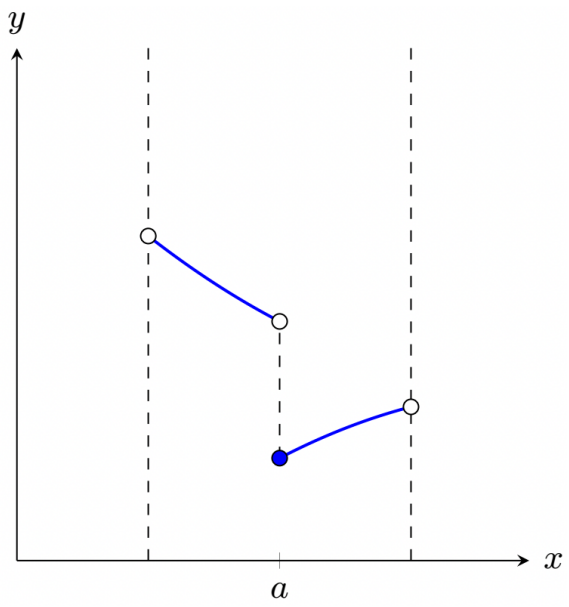
for $a = -1, 1, 2,$ and 3 separately.

Where do you think this function is discontinuous? Write your definition of a discontinuous function.

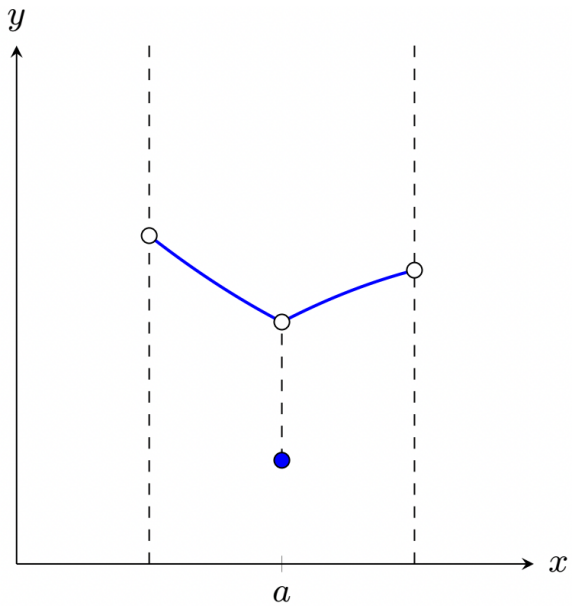
Catalog of basic discontinuity types



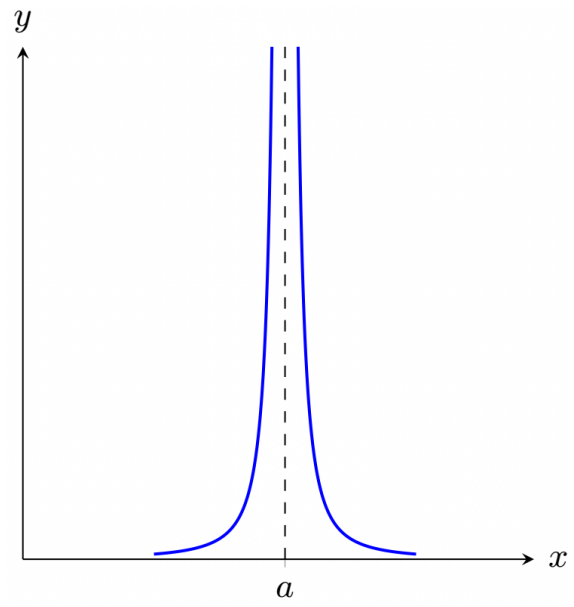
Removable Discontinuity



Non-removable Discontinuity



Removable Discontinuity



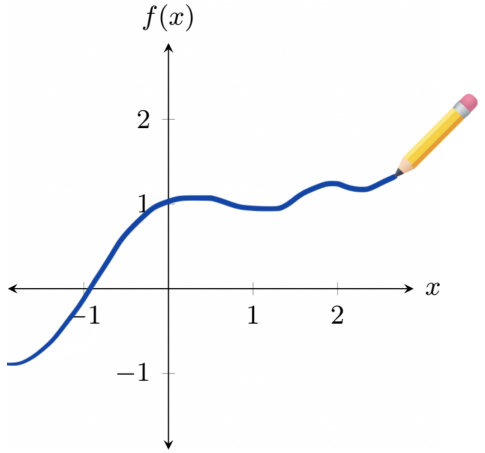
Non-removable Discontinuity

A discontinuity is called removable if it can be made continuous by appropriately defining or redefining $f(a)$

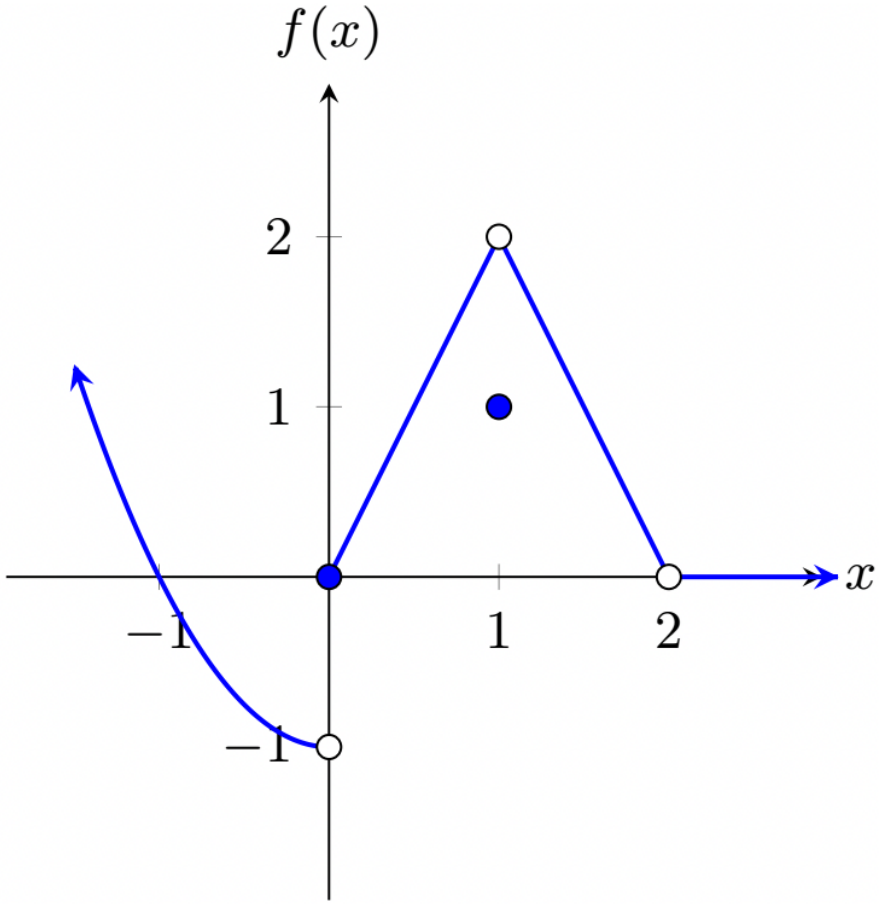
Definition of Continuity

A function is continuous at a if the following three condition are met.

1. $f(a)$ is defined.
2. $\lim\limits_{x \to a} f(x)$ exists.
3. $\lim\limits_{x \to a} f(x) = f(a)$



A function that is continuous on an open interval satisfies these three conditions at each point on that interval.



🤔 At which numbers is f discontinuous? Why?

Provide a reason for each discontinuity by highlighting the condition for continuity that is not satisfied.

🔍 Spot discontinuities without a graph

Where are each of the following functions discontinuous? How do you know?

1. $f(x) = \dfrac{1}{x}$

2. $y = \dfrac{x^2 - 1}{x^2 - 4x + 3}$

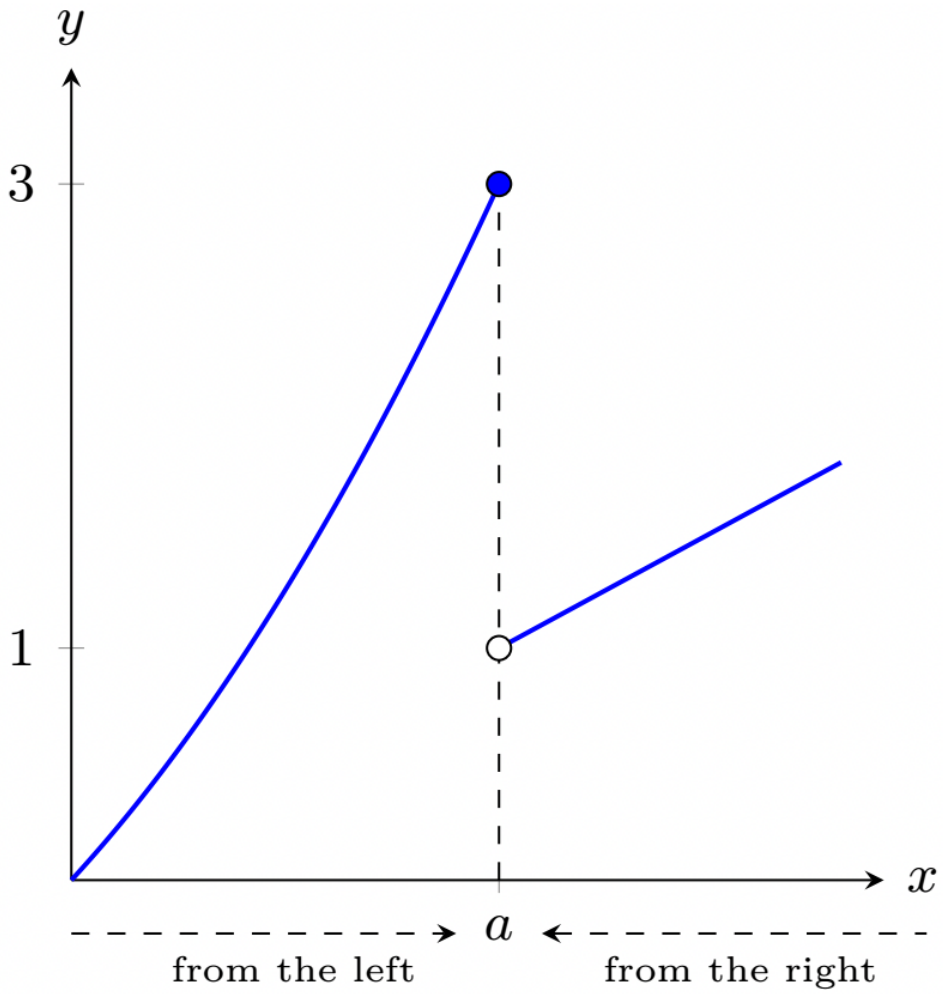
One-Sided Limits

Limit of $f(x)$ as $x \to a$ from the right of a

$$\lim_{x \to a^+} f(x)$$

Limit of $f(x)$ as $x \to a$ from the left of a

$$\lim_{x \to a^-} f(x)$$

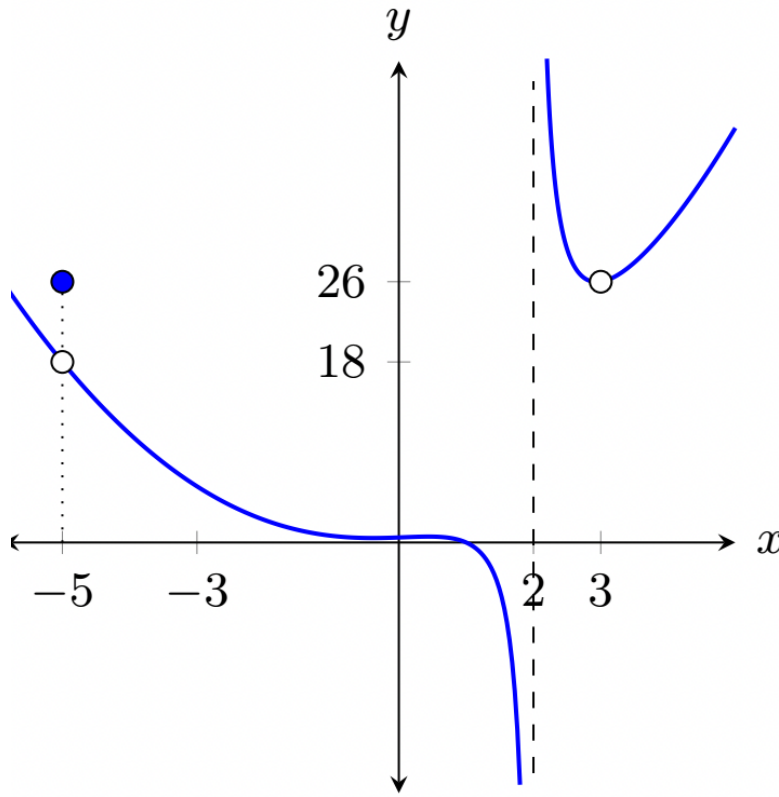


One-Sided and Two-Sided Limits Relationship

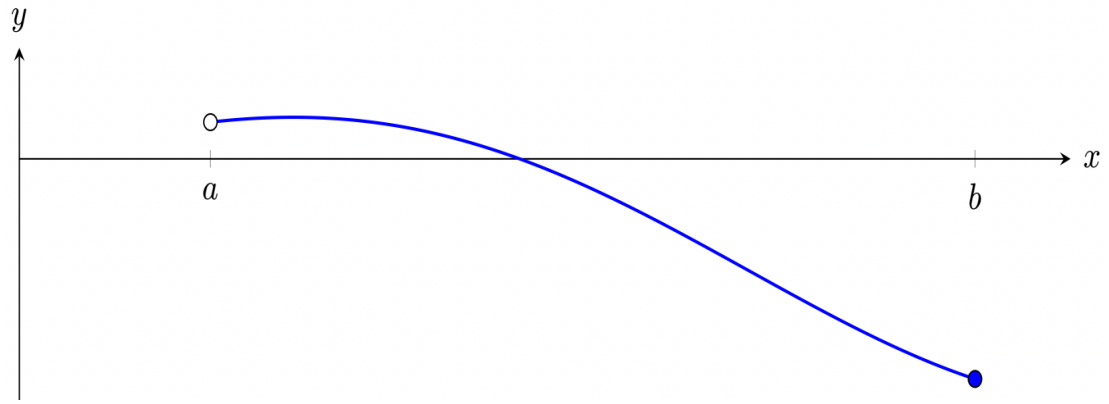
A function $f(x)$ has a limit as x approaches a if and only if the right-hand and left-hand limits exist at a and are equal.

$$\lim_{x \to a} f(x) = L \iff \lim_{x \to a^+} f(x) = L \quad \text{and} \quad \lim_{x \to a^-} f(x) = L$$

Consider the single-sided vs double-sided limits at $x = -5, 2, \text{and } 3$ for the function whose graph is shown on the right.



Continuity at the endpoints



🤔 How would we check the continuity conditions at b?

Properties of Continuity

If f and g are continuous at $x = a$, then the following combinations of these functions are also continuous at $x = a$.

1. $f \pm g$ Sums and differences of continuous functions are continuous. 🤔 Where?
2. $f \cdot g$
3. $k \cdot g$, where k is a constant
4. $\frac{f}{g}$, provided $g(a) \neq 0$

Consequence of Properties of Continuity

The following types of functions are continuous at every number in their domains.

Polynomials	Rational Functions	Root Functions	Trigonometric Functions

Determine the intervals on which each of the following functions are continuous?

1. $h(x) = \dfrac{x^2 + 2x + 16}{x^2 - 1}$

2. $f(x) = \sqrt{x} + \dfrac{x+1}{x-1} - \dfrac{x+1}{x^2-1}$

! Verify continuity at a point.

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$

Is $f(x)$ continuous at $x = 1$? Write a reasoned response using the 3 conditions of continuity.

Find $x = a$ that would make the given function continuous.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 3 \\ 2ax & \text{if } x \geq 3 \end{cases}$$