What is the Intermediate Value Theorem?

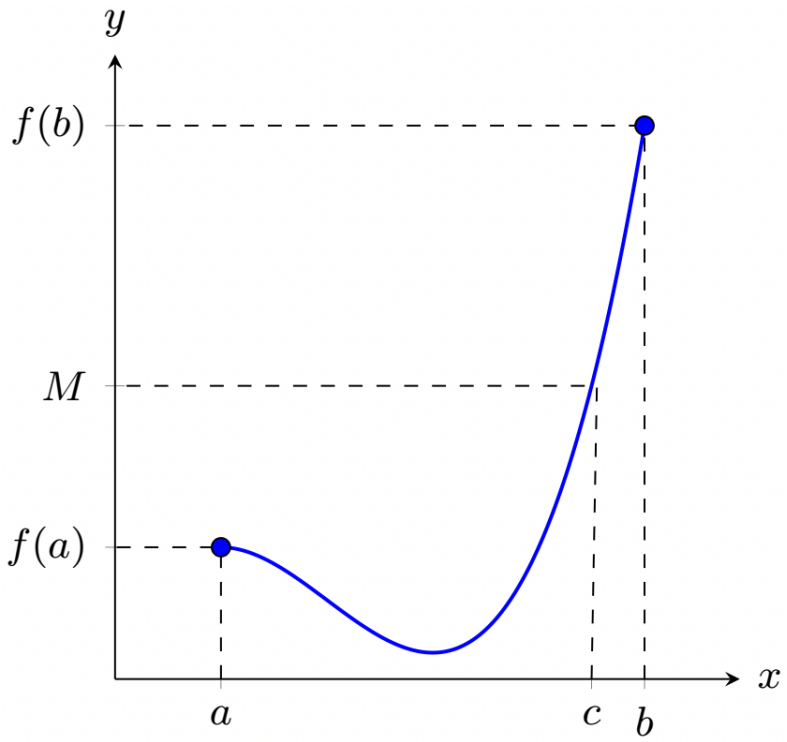
Quick Check

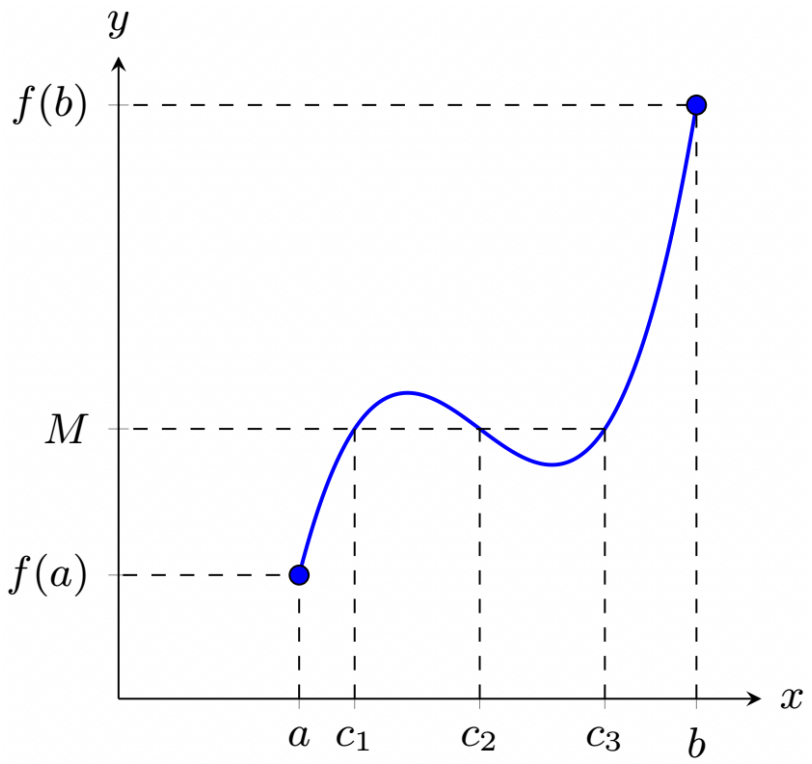
Show that $f(x) = x^5 + 2x^3 + x - 5$ has at least one real solution (an x-value for which $f(x) = 0$).

You don't have to find the solution itself; you must provide reasoning for the existence of one.

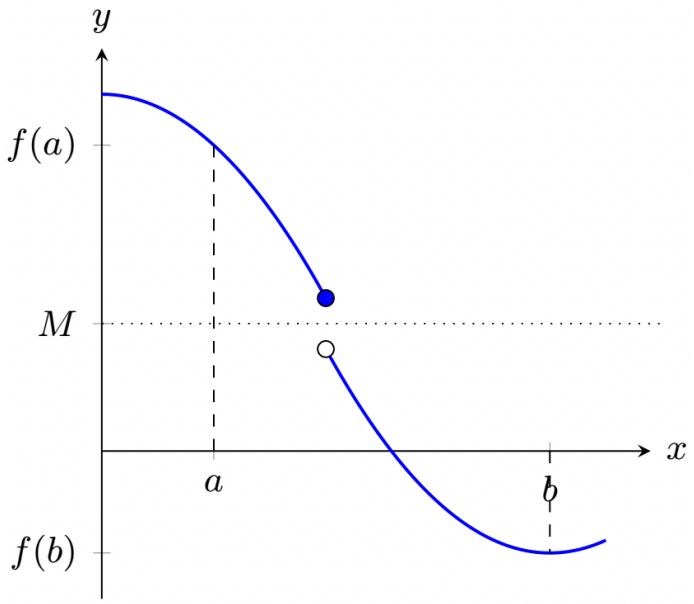
Existence Theorem - Intermediate Value Theorem (IVT)

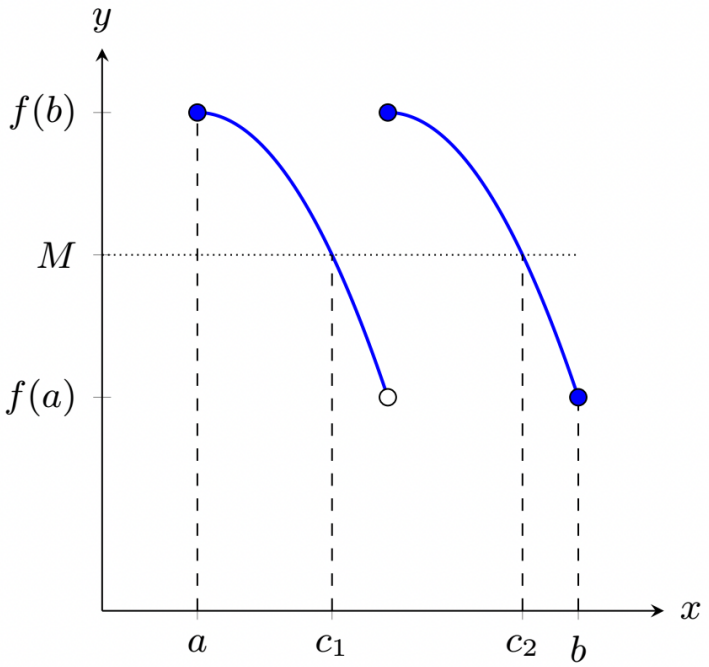
If f is a continuous function on a closed interval $[a, b]$ and M is any number between $f(a)$ and $f(b)$, inclusive, then there is at least one number c in $[a, b]$ such that $f(c) = M$.





A question of existence





🤔 Is continuity a necessary condition?

IVT used to show existence of roots (x-intercepts, solutions to $f(x) = 0$).

Show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero in $[0, 1]$.

IVT Applications

1. Explain why $f(x) = x^2 - 2 - \cos(x)$ has a zero in the interval $[0, \pi]$.
2. Given $f(x) = \dfrac{\sin(x)}{x^2 - 12}$. Prove that $f(x) = 4$ for some x in $[0, 3.46]$.
3. Ask your calculator for zero of $f(x) = x^4 - 4x^3$ between $x = 1$ and $x = 2$. What does it tell you? Explain.