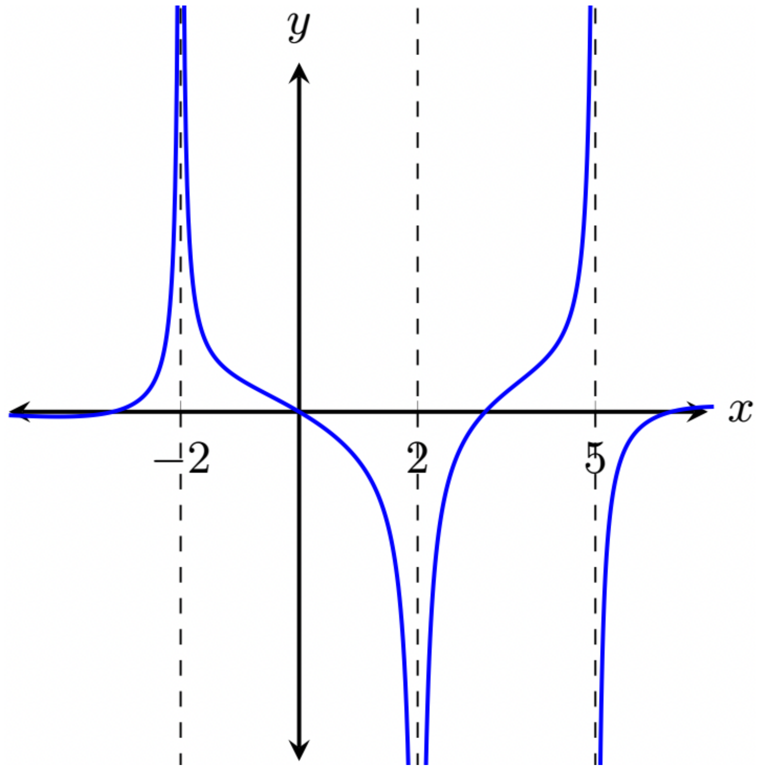
What sort of behavior does an infinite limit indicate for a function?

Quick Check

Draw sketches of functions to showcase various possibilites under which $\lim_{x \to a} f(x)$ does not exist. Each sketch should focus on one case.

Infinite Limit

A limit in which $f(x)$ increases or decreases without bound as x approaces a.



Describe the function behavior for each limit.

$$\lim_{x \to 5^-} f(x)$$

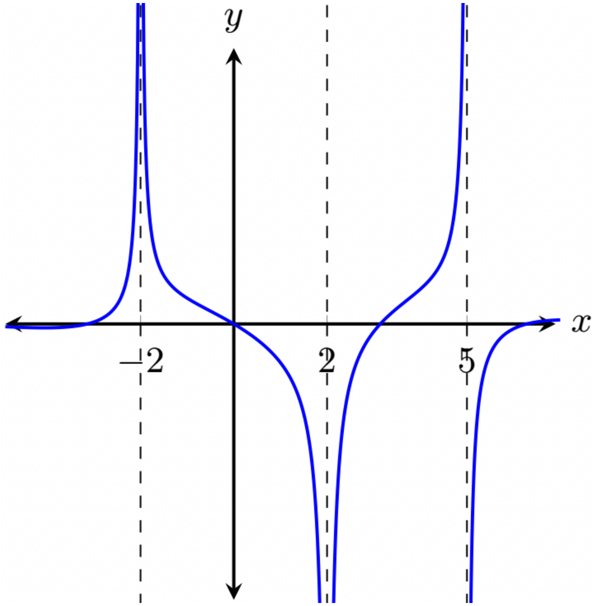
$$\lim_{x \to 5^+} f(x)$$

$$\lim_{x \to 5} f(x)$$

⚠️ Is ∞ a number? Why are we using this symbol?

$$\lim_{x \to a} f(x) = \infty$$

This does NOT mean that we are regarding ∞ or $-\infty$ as a number. Nor does it mean that the limit exists. We are using ∞ or $-\infty$ to indicate unbounded behavior.



Find each limit.

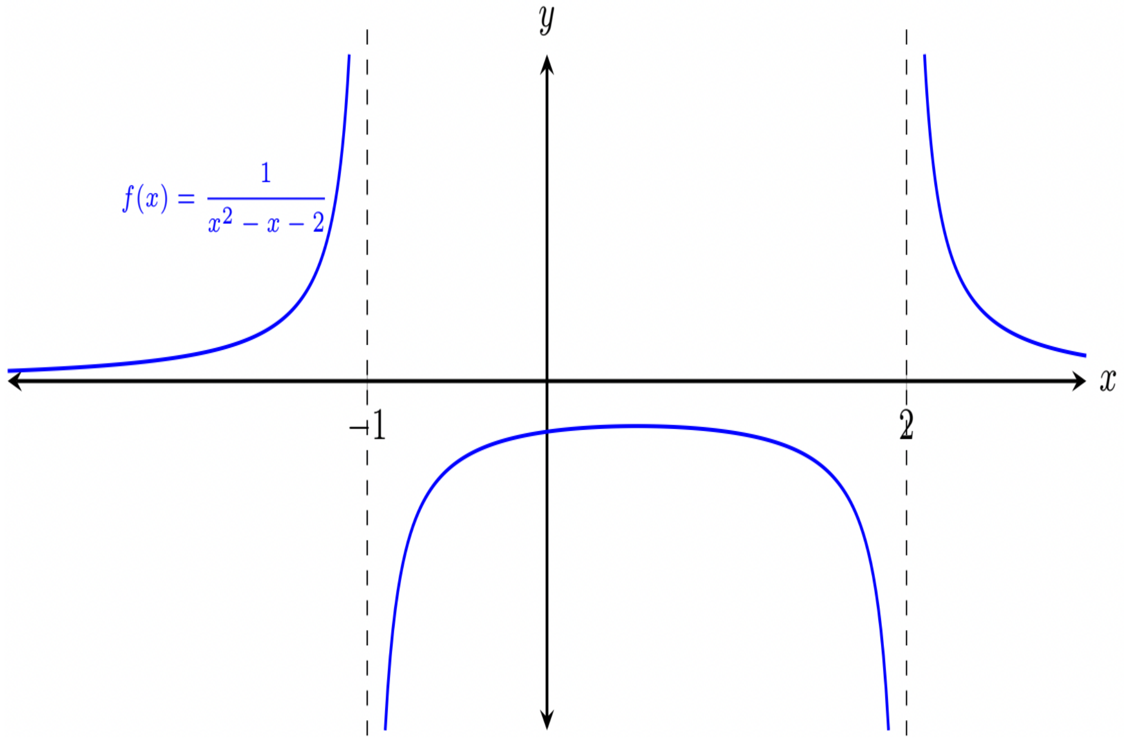
$$\lim_{x \to -2^-} f(x) \qquad \lim_{x \to 2^-} f(x)$$

$$\lim_{x \to -2^+} f(x) \qquad \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to -2} f(x) \qquad \lim_{x \to 2} f(x)$$

Infinite Limits indicate vertical asymptotes

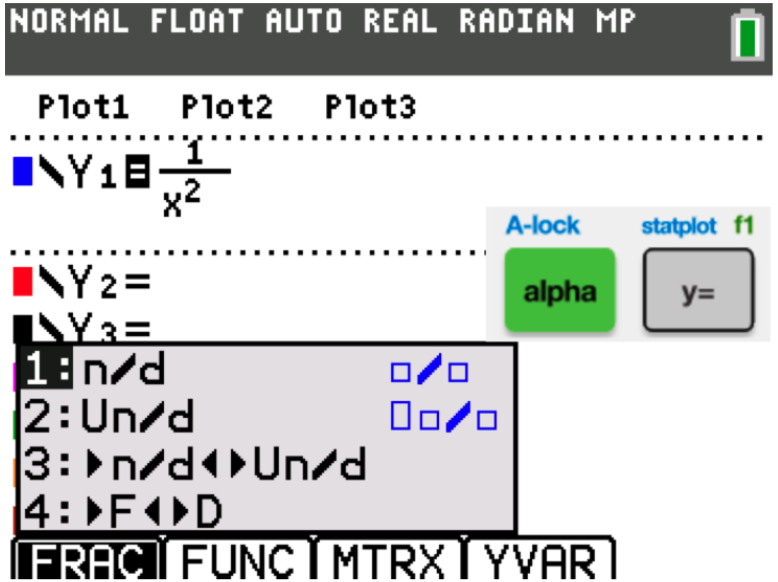
A vertical line $x = a$ is called a vertical asymptote if $f(x)$ approaces infinity or negative infinity as $x \to a$ from the right or the left or both.

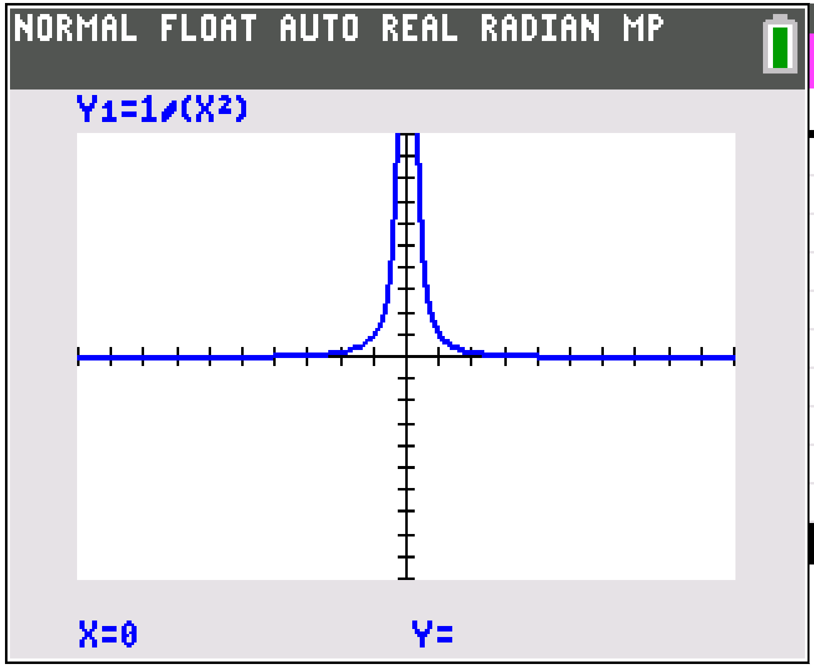


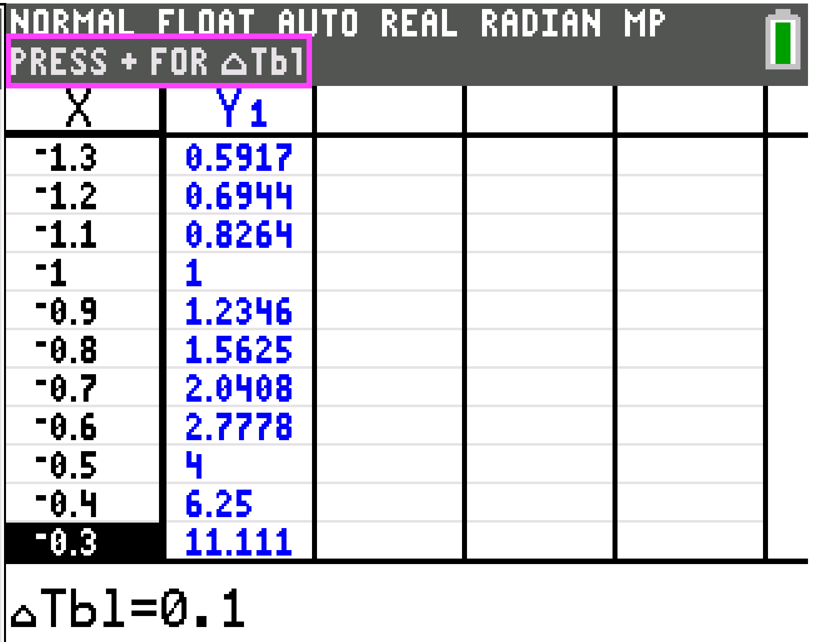
1. $\lim_{x \to -1^-} f(x)$
2. $\lim_{x \to -1^+} f(x)$
3. $\lim_{x \to 2^-} f(x)$
4. $\lim_{x \to 2^+} f(x)$

What are the equations of vertical asymptotes.

🔍 Unbounded Behavior. Why? And big number challenge.







🤔 What is the connection between vertical asymptotes and denominators of functions?

Identify vertical asymptotes without a graph

1. $f(x) = \dfrac{5}{x-2}$	2. $f(x) = \dfrac{x^2+2x-8}{x^2-4}$	3. $f(x) = \cot(x)$

Determining infinite limits

1. $\lim_{x \to 1^+} \frac{2+x}{1-x}$

2. $\lim_{x \to 2^+} \frac{5}{(x-2)^3}$

3. $\lim_{x \to 3^-} \frac{x^2}{(x^2-9)}$

🤨 Remember ∞ is a shorthand notation

Let $number > 0$

$$\infty \pm number \to \infty$$

$$\infty \cdot number \to \infty$$

$$\frac{number}{\to \infty} \to 0$$

Find each limit (if it exists).

1. $\displaystyle\lim_{x \to 0^-} x^2 - \frac{1}{x}$
2. $\displaystyle\lim_{x \to \pi^-} \frac{9999999999999999999}{\csc(x)}$
3. $\displaystyle\lim_{x \to 0^+} 3 \cdot \cot(x)$
4. $\displaystyle\lim_{x \to 3} \frac{x-2}{x^2}$