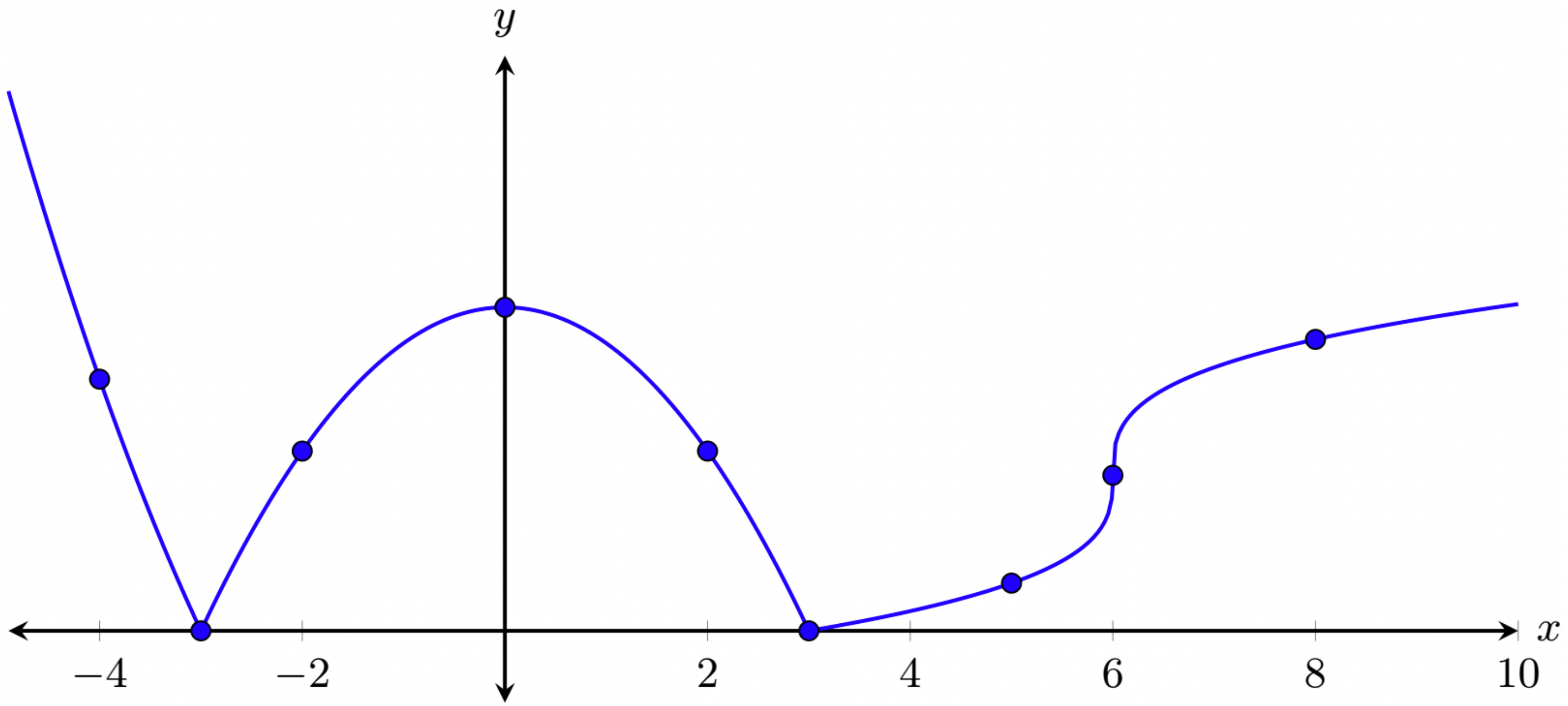
What is meant by "slope of the curve at the given point"?

Quick Check

1. Explain how you could find the equation of the line tangent to the circle $x^2 + y^2 = 169$ at the point $(5, 12)$. Start by drawing out the circle, the point, and the tangent at that point.

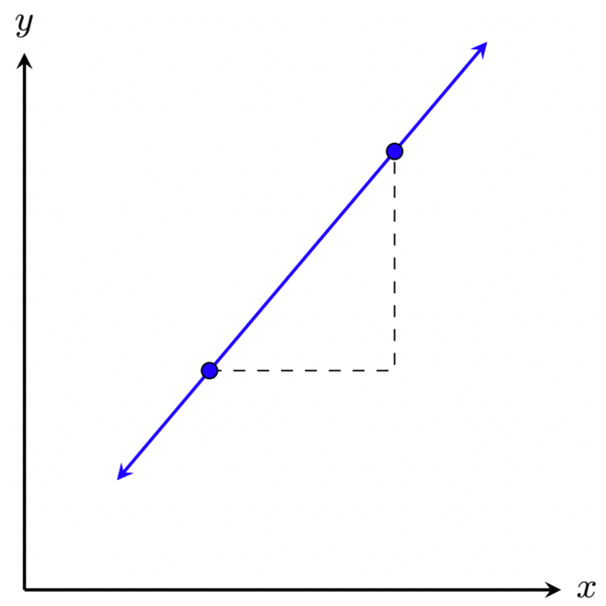
2. Write down your definition of a tangent line?

Draw a tangent line to the curve at the given point(s).



🔍 Did your written definition of a tangent work in the context of general curves? Explain.

Equation of a line

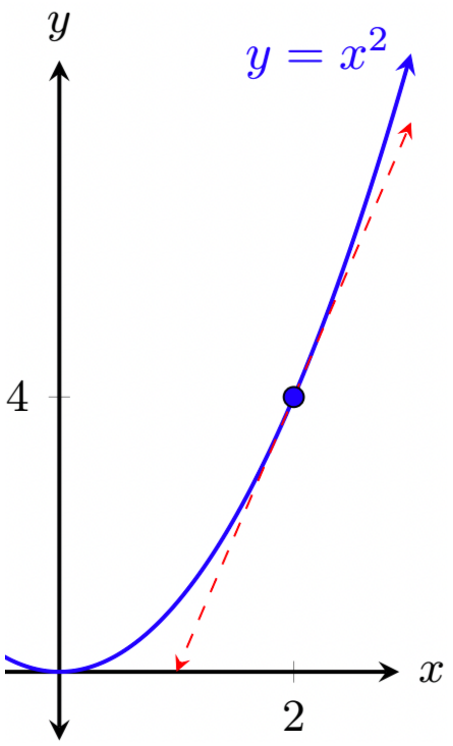


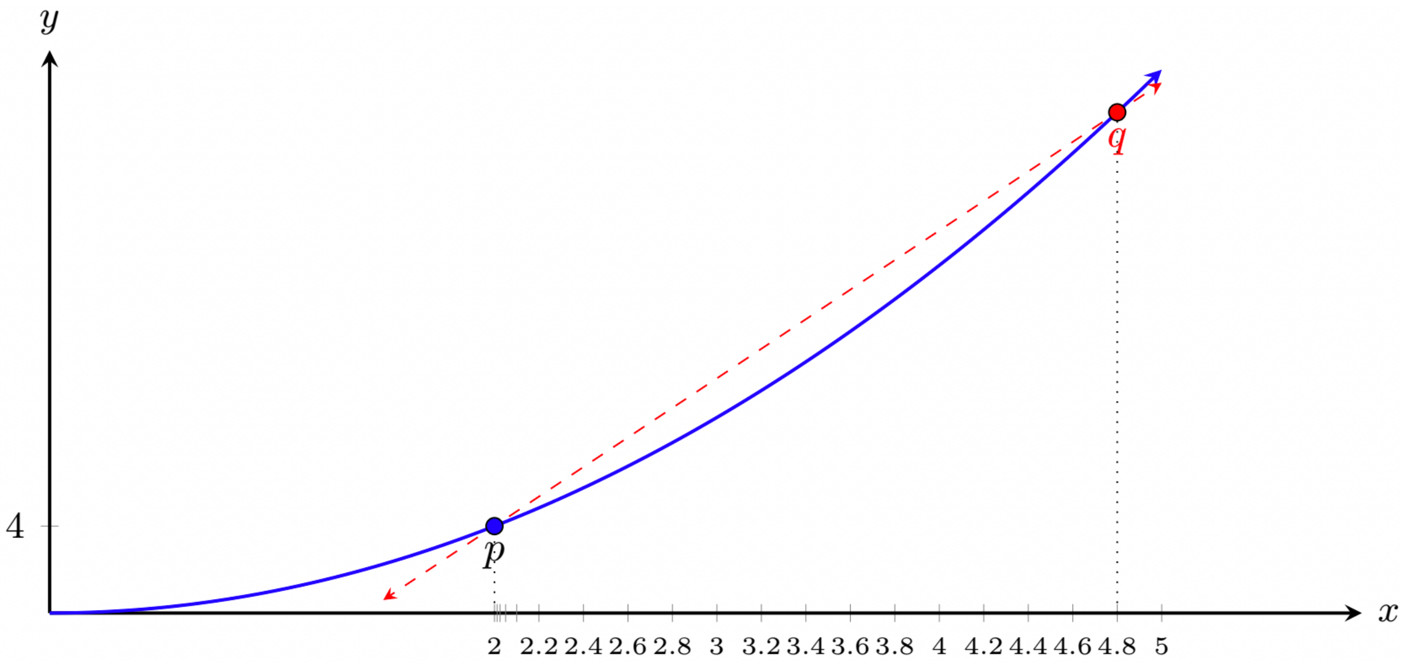
A line is defined by 2 points.

To write the equation of a line we need: 2 Points OR Point + Slope

Point-Slope Form of the equation of the line: $y - y_1 = m \cdot (x - x_1)$

The Tangent Problem





Approximate slope of the tangent line through the given point with slope of a secant line. Make the approximation better by finding the secant slope on a smaller and smaller interval.

Secant to Tangent

$$m_{secant} = \frac{\Delta y}{\Delta x}$$

Interval p to q	$2 - 5$	$2 - 4.8$	...	$2 - 2.09$	...
Secant Slope Difference Quotient	$\frac{f(5)-f(2)}{5-2}$	$\frac{f(4.8)-f(2)}{4.8-2}$	...	$\frac{f(2.09)-f(2)}{2.09-2}$	...

We ask what slope value would we get if the interval were to get infinitely small.

🌐 Geogebra - The Tangent Problem Animation

Slope of the curve

The slope $m_{tangent}$ of the tangent line to $y = f(x)$ at a point $\big(x_0, f(x_0)\big)$ is given by

$$m_{tangent} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

provided this limit exists.

🤔 What if this limit is an infinite limit ($= \pm\infty$) ?

Test the formula with a simple case

Find the slope of the graph of $f(x) = 3x + 1$ at the point $(2, 7)$.

Test the reasonableness of your answer against a graph

1. Find the slopes of tangent line to the graph of $f(x) = x^2 + 4x$ at $x = 0, 2, \text{and } 4.$

2. Find the expression that provides the slopes of the curve $y = \dfrac{1}{x}$ at any point $x = a \neq 0$. Then, find the $x - value$ where slope of the curve is $\dfrac{-1}{4}$.