## What is meant by "slope of the curve at the given point"?

## Quick Check

1. Explain how you could find the equation of the line tangent to the circle $x^{2}+y^{2}=169$ at the point $(5,12)$. Start by drawing out the circle, the point, and the tangent at that point.
2. Write down your definition of a tangent line?

## Draw a tangent line to the curve at the given point(s).


O. Did your written definition of a tangent work in the context of general curves? Explain.

## Equation of a line



A line is defined by 2 points.

To write the
equation of a 2 Points OR Point + Slope line we need:

Point-Slope Form of the equation of the line: $y-y_{1}=m \cdot\left(x-x_{1}\right)$

## The Tangent Problem




Approximate slope of the tangent line through the given point with slope of a secant line. Make the approximation better by finding the secant slope on a smaller and smaller interval.

## Secant to Tangent

$$
m_{\text {secant }}=\frac{\Delta y}{\Delta x}
$$

| Interval |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| p to q | $2-5$ | $2-4.8$ | $\ldots$ | $2-2.09$ | $\ldots$ |
| Secant Slope | $\frac{f(5)-f(2)}{5-2}$ | $\frac{f(4.8)-f(2)}{4.8-2}$ | $\ldots$ | $\frac{f(2.09)-f(2)}{2.09-2}$ | $\ldots$ |
| Difference Quotient |  |  |  |  |  |

We ask what slope value would we get if the interval were to get infinitely small.
Geogebra - The Tangent Problem Animation

## Slope of the curve

The slope $m_{\text {tangent }}$ of the tangent line to $y=f(x)$ at a point $\left(x_{0}, f\left(x_{0}\right)\right)$ is given by

$$
m_{\text {tangent }}=\lim _{h \rightarrow 0} \frac{f\left(x_{0}+h\right)-f\left(x_{0}\right)}{h}
$$

provided this limit exists.
i. What if this limit is an infinite limit $(= \pm \infty)$ ?

## Test the formula with a simple case

Find the slope of the graph of $f(x)=3 x+1$ at the point $(2,7)$.

## Test the reasonableness of your answer against a graph

1. Find the slopes of tangent line to the graph of $f(x)=x^{2}+4 x$ at $x=0,2$, and 4 .
2. Find the expression that provides the slopes of the curve $y=\frac{1}{x}$ at any point $x=a \neq 0$. Then, find the $x-$ value where slope of the curve is $\frac{-1}{4}$.
