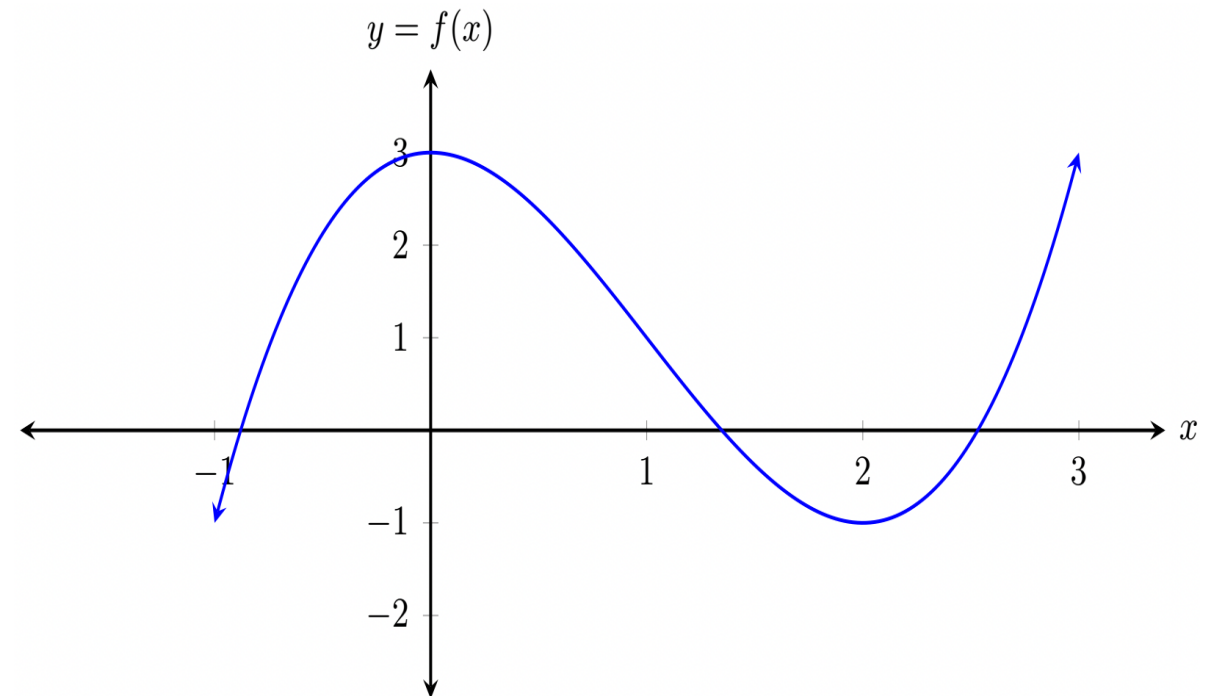


What are the interpretations of the limit of the difference quotient?

Quick Check

For the function shown in the sketch, give the intervals or points on the x - *axis* where the slope of the curve is

1. Positive
2. Negative
3. Zero



Derivative

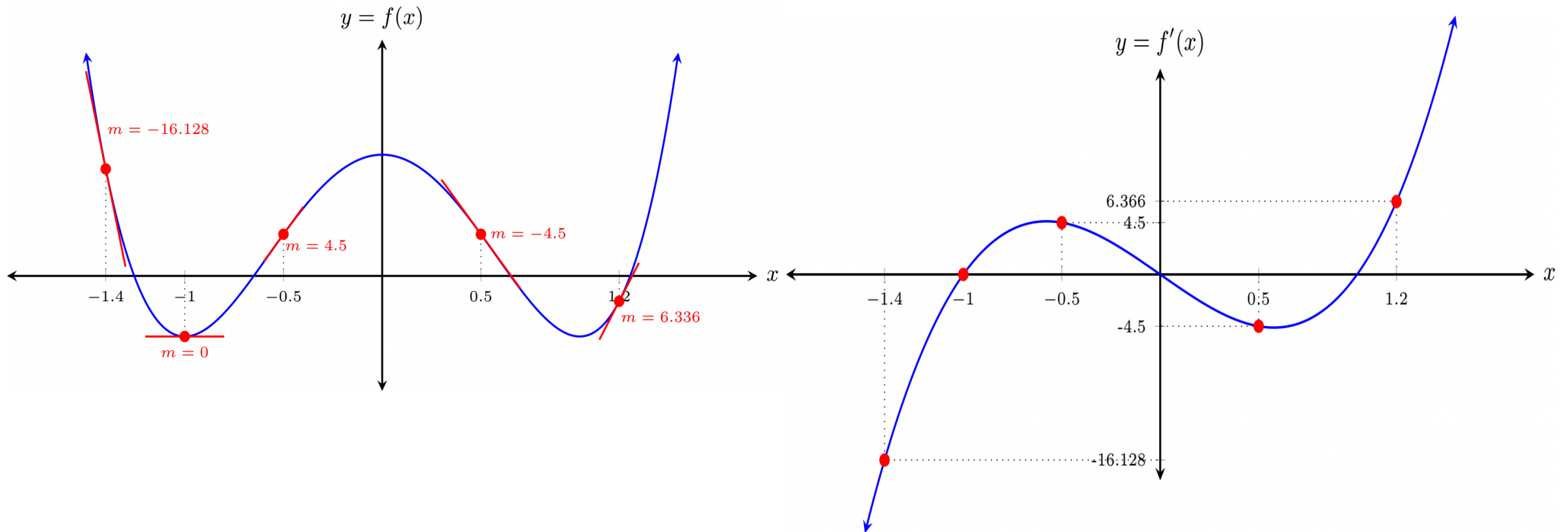
The following are all interpretations for the limit of the difference quotient at the point $(x_0, f(x_0))$

$$\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

1. Slope of the graph of $y = f(x)$ at the point on the graph.
2. Slope of the tangent line to the curve $y = f(x)$ at the point.
3. Instantaneous rate of change of $f(x)$ with respect to x at the point.
4. Derivative $f'(x_0)$ at the point.

The Derivative Function

The derivative function gives the slope of the tangent line to the graph of $f(x)$ at the point $(x, f(x))$ provided that the graph has a tangent at this point.



Important Vocabulary

Differentiation is the process of finding the derivative of a function.

A function is **differentiable** at x if its derivative exists at x and is differentiable on an open interval if its derivative exists at every point in the interval.

Notation:

f' - derivative of the function f

$f'(x)$ - derivative of the function f with respect to x

$h'(t)$ - derivative of the function h with respect to t

$\frac{dy}{dx}$ - derivative of y with respect to x

$\frac{df}{dt}$ - derivative of f with respect to t

Derivative using the limit process

Find the derivative of $f(x) = 1 - x^2$. Use the derivative function to find the slope of the tangent line to $f(x)$ at the points $(-2, 3)$, $(0, 1)$, and $(1, 0)$.

Pascal's Triangle and the Binomial Theorem

			1			
		1		1		
	1		2		1	
	1	3		3	1	
	1	4	6	4	1	
1	5	10	10	5	1	
1	6	15	20	15	6	1

$$(a + b)^0 = 1$$

$$(a + b)^1 = a + b$$

$$(a + b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a + b)^5 = \dots$$

$$(a + b)^6 = \dots$$

Expand $(x + h)^5$.

Find $\frac{df}{dx}$ using the limit process. Use the derivative to find the equation of the tangent line to the graph of f at $(2, 8)$.

$$f(x) = x^3$$

🤔 Let me graph the function and its tangent. Does the answer pass the visual check?

Find the derivative using the limit process

1. $f(x) = x^3 + 2x$. Find $f'(x)$.
2. $y = \sqrt{x}$. Find $\frac{dy}{dx}$.
3. $h(t) = \frac{4}{t}$. Find $h'(t)$. Find and explain the meaning of $h'(1)$ and $h'(10)$. Sketch to illustrate both.
4. $g(x) = x^2 + 1$. Find $\frac{dg}{dx}$. Sketch the graph of the function and its derivative side by side. On what intervals is $g'(x)$ positive or negative? What do you notice about $g(x)$ on the same intervals.
5. $f(x) = \frac{1}{x+1}$. Find the equation of the tangent line to $f(x)$ at the point $(0, 1)$.