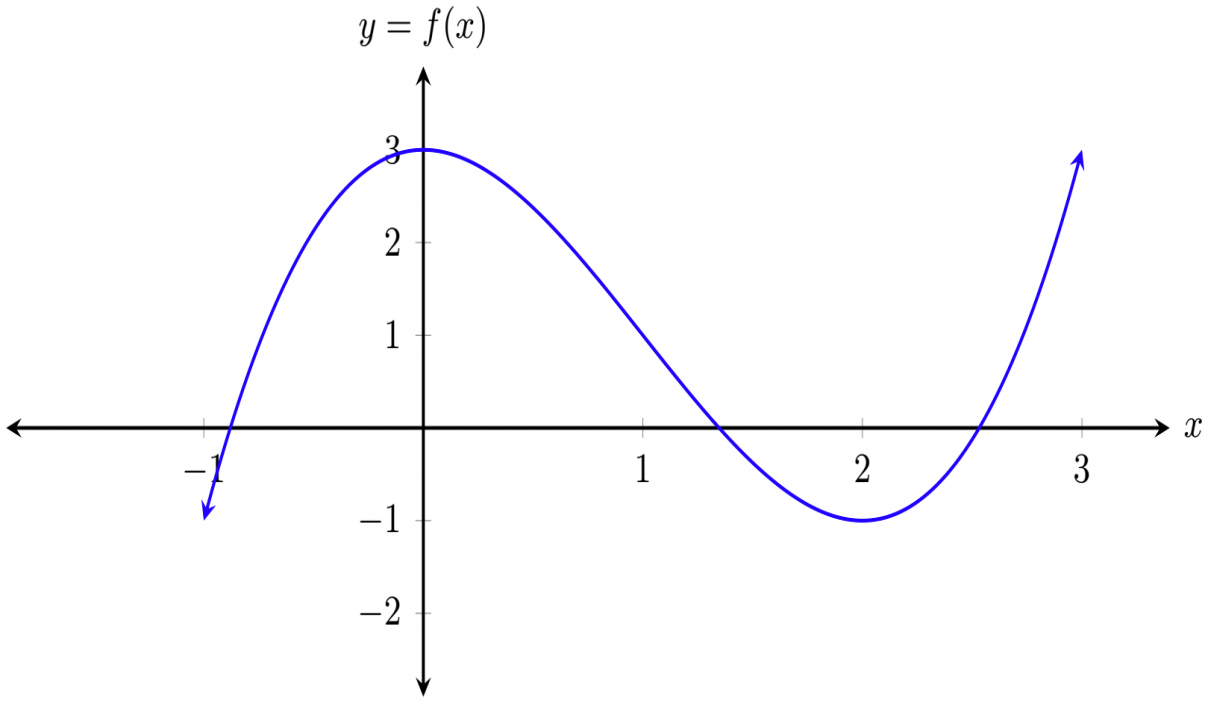
What are the interpretations of the limit of the difference quotient?

Quick Check

For the function shown in the sketch, give the intervals or points on the $x-axis$ where the slope of the curve is

1. Positive
2. Negative
3. Zero



🤔 Derivative

The following are all interpretations for the limit of the difference quotient at the point $\big(x_0, f(x_0)\big)$

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

1. Slope of the graph of $y = f(x)$ at the point on the graph.

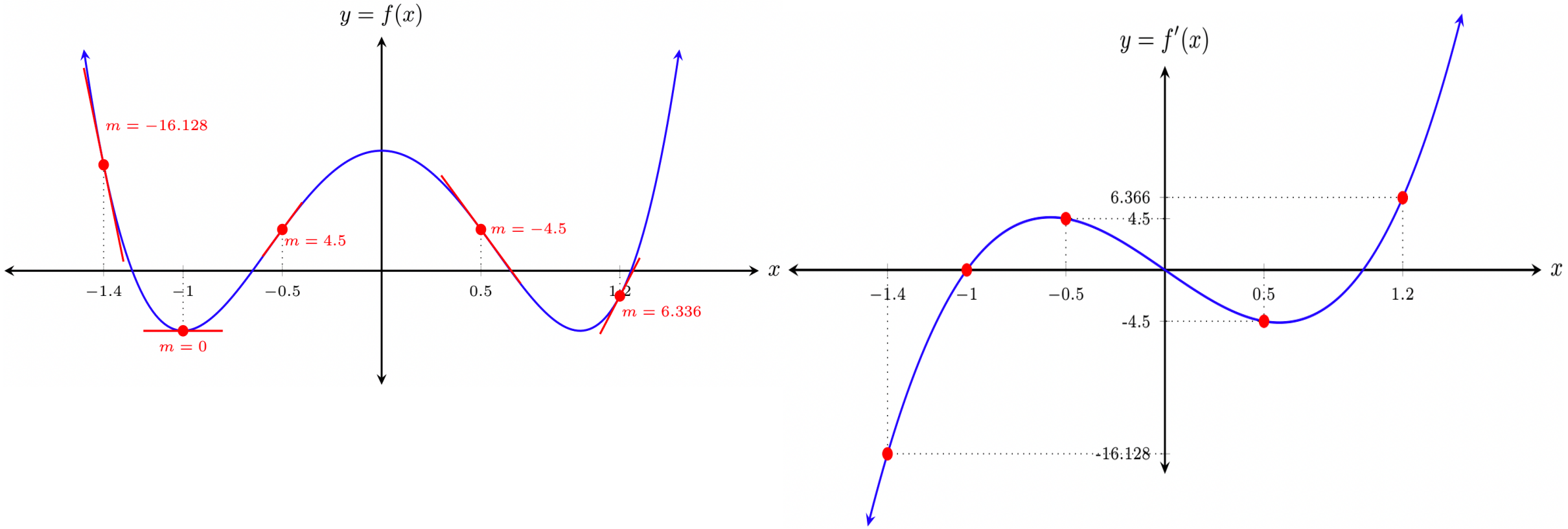
2. Slope of the tangent line to the curve $y = f(x)$ at the point.

3. Instantaneous rate of change of $f(x)$ with respect to x at the point.

4. Derivative $f'(x_0)$ at the point.

The Derivative Function

The derivative function gives the slope of the tangent line to the graph of $f(x)$ at the point $(x, f(x))$ provided that the graph has a tangent at this point.



Important Vocabulary

Differentiation is the process of finding the derivative of a function.

A function is differentiable at x if its derivative exists at x and is differentiable on an open interval if its derivative exists at every point in the interval.

Notation:

f' - derivative of the function f

$f'(x)$ - derivative of the function f with respect to x

$h'(t)$ - derivative of the function h with respect to t

$\frac{dy}{dx}$ - derivative of y with respect to x

$\frac{df}{dt}$ - derivative of f with respect to t

Derivative using the limit process

Find the derivative of $f(x) = 1 - x^2$. Use the derivative function to find the slope of the tangent line to $f(x)$ at the points $(-2, 3), (0, 1), \text{and } (1, 0)$.

Pascal's Triangle and the Binomial Theorem

$$
\begin{array}{ccccccccccc}
 & & & & 1 & & & & & & \\
 & & & 1 & & 1 & & & & & \\
 & & 1 & & 2 & & 1 & & & & \\
 & 1 & & 3 & & 3 & & 1 & & & \\
1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

$$
\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a+b \\
(a+b)^2 &= 1a^2+2ab+1b^2 \\
(a+b)^3 &= 1a^3+3a^2b+3ab^2+b^3 \\
(a+b)^4 &= 1a^4+4a^3b+6a^2b^2+4ab^3+b^4 \\
(a+b)^5 &= \dots \\
(a+b)^6 &= \dots
\end{aligned}
$$

Expand $(x+h)^5$.

Find $\frac{df}{dx}$ using the limit process. Use the derivative to find the equation of the tangent line to the graph of f at $(2, 8)$.

$f(x) = x^3$

🤔 Let me graph the function and its tangent. Does the answer pass the visual check?

Find the derivative using the limit process

1. $f(x) = x^3 + 2x$. Find $f'(x)$.
2. $y = \sqrt{x}$. Find $\dfrac{dy}{dx}$.
3. $h(t) = \dfrac{4}{t}$. Find $h'(t)$. Find and explain the meaning of $h'(1)$ and $h'(10)$. Sketch to illustrate both.
4. $g(x) = x^2 + 1$. Find $\dfrac{dg}{dx}$. Sketch the graph of the function and its derivative side by side. On what intervals is $g'(x)$ positive or negative? What do you notice about $g(x)$ on the same intervals.
5. $f(x) = \dfrac{1}{x+1}$. Find the equation of the tangent line to $f(x)$ at the point $(0, 1)$.