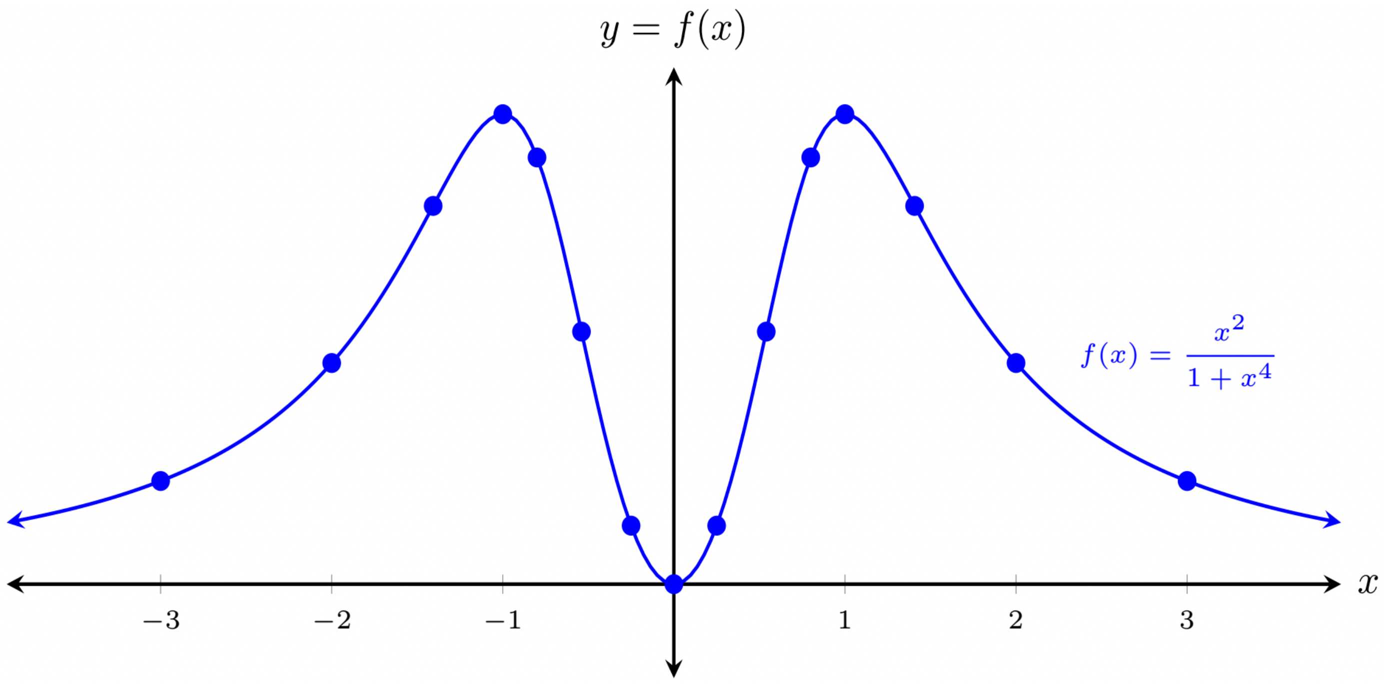
Does differentiability imply continuity and vice versa?

Quick Check

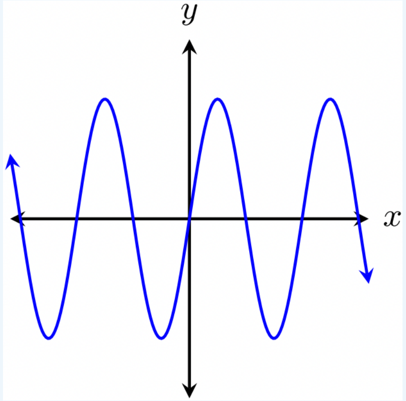


1. Draw a tangent segment at each highlighted point.
2. What information do the tangents/slopes give about $f(x)$?
3. Positive, Negative or Zero ?

$f'(-3)$	$f'(0)$	$f'(2)$
$f'(-1.5)$	$f'(.5)$	$f'(50)$

Match the graph of each function with the graph of its derivative. Reason?

1.



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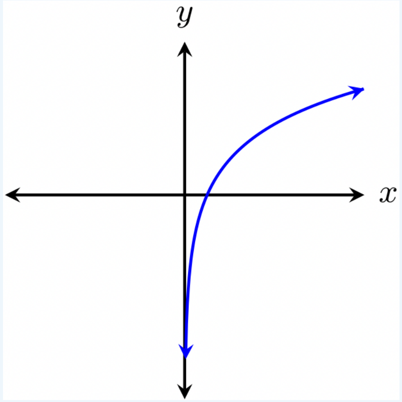

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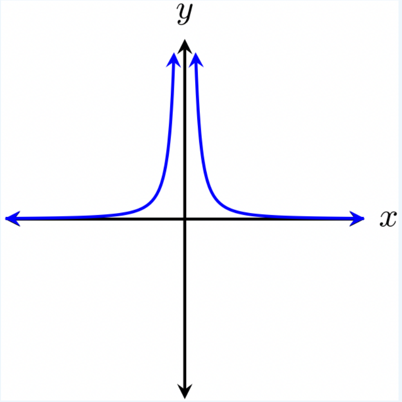
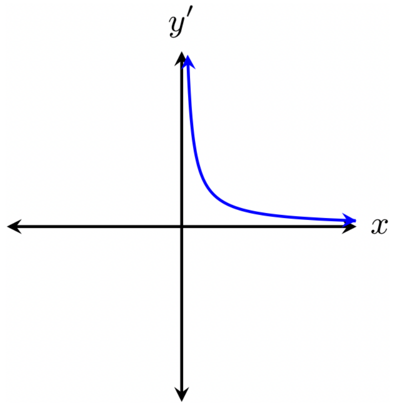
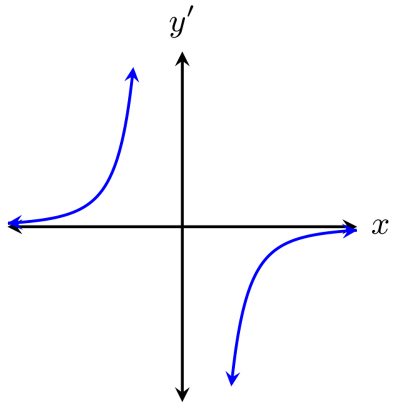


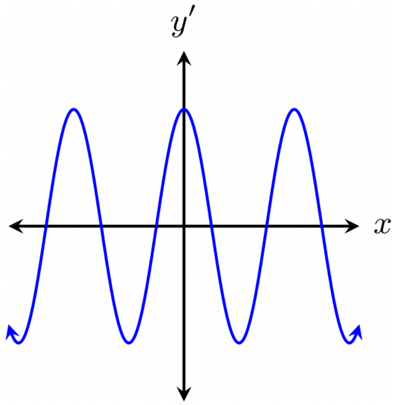
a.



b.



c.



When can a function fail to be differentiable?

Recall that we say $f(x)$ is differentiable at some point (x_0, y_0) if

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

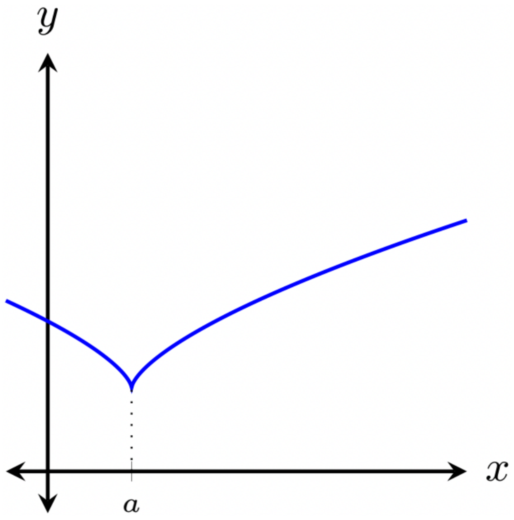
exists at $x = x_0$.

❗ The function needs to be defined at the point for us to talk about its derivate there.

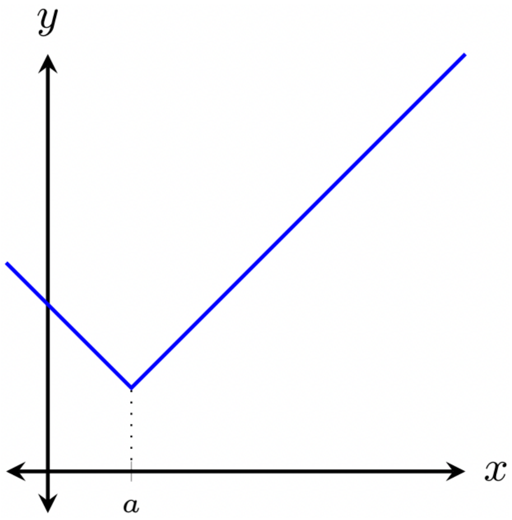
What does this imply for where a function may not be differentiable?

🤔 How can a limit... the above limit... fail to exist?

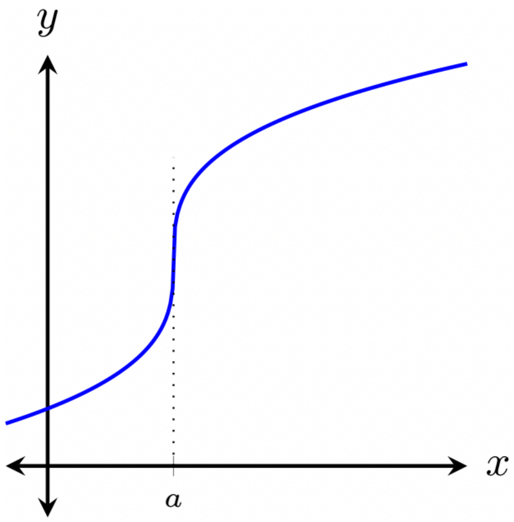
Ways for a function not to be differentiable at a point



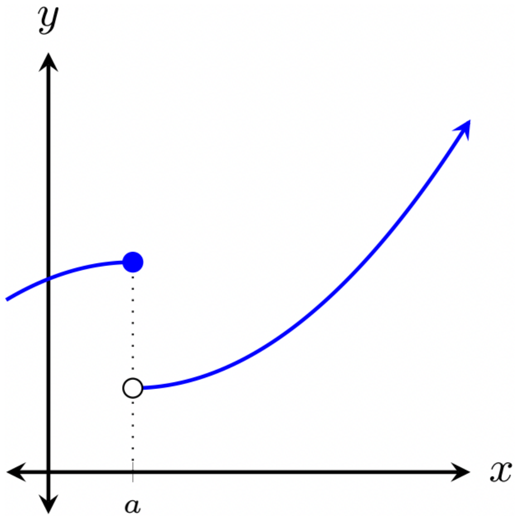
Cusp



Sharp Turn

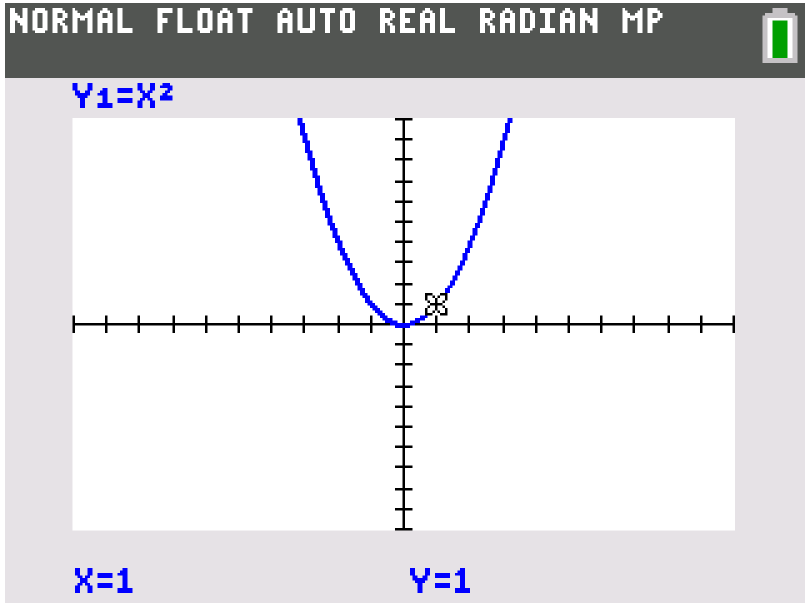


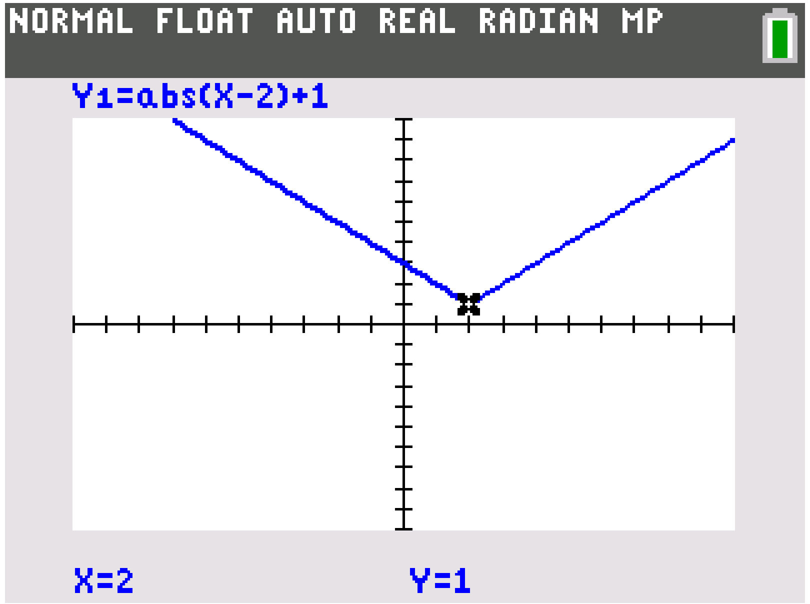
Vertical Tangent



Discontinuity

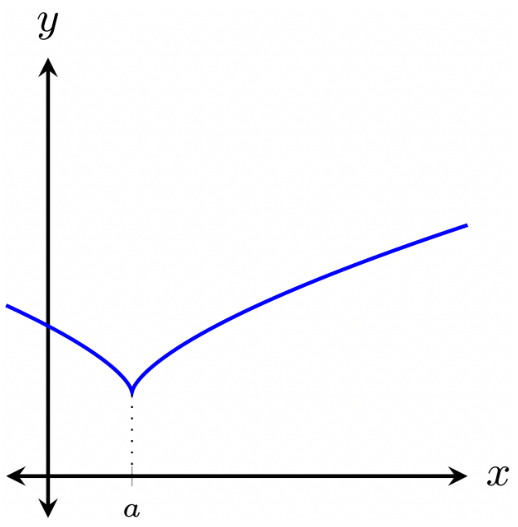
🔍 Zoooom away - Differentiability and Smoothness





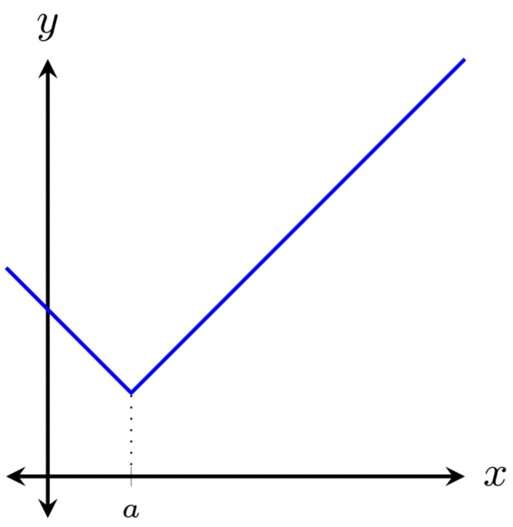
Zoom in at the indicated point on your calculator.
What do you notice after after several ZOOM INs?

Visual Understanding



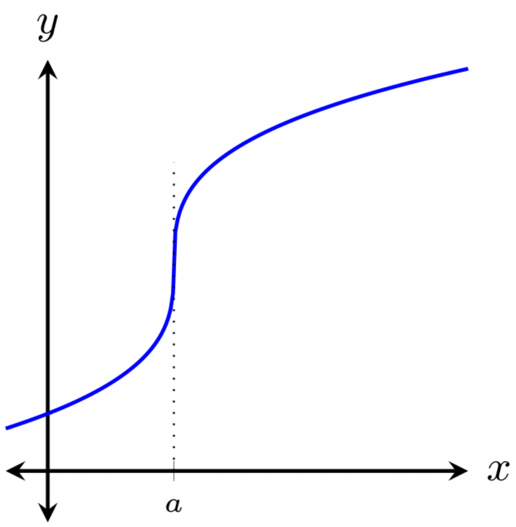
Cusp

🌐 Geogebra Animation



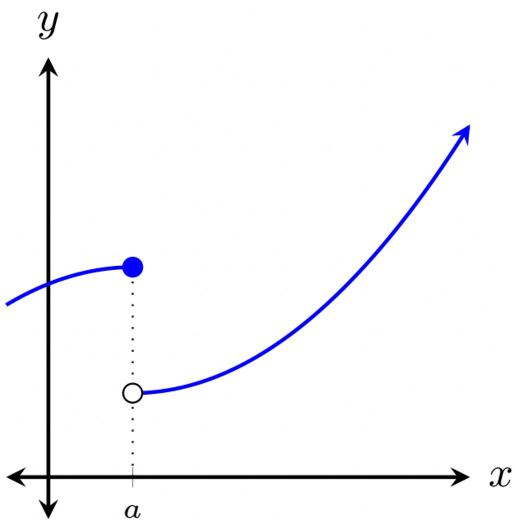
Sharp Turn

🌐 Geogebra Animation



Vertical Tangent

🌐 Geogebra Animation



Discontinuity

🌐 Geogebra Animation

Differentiability and Continuity

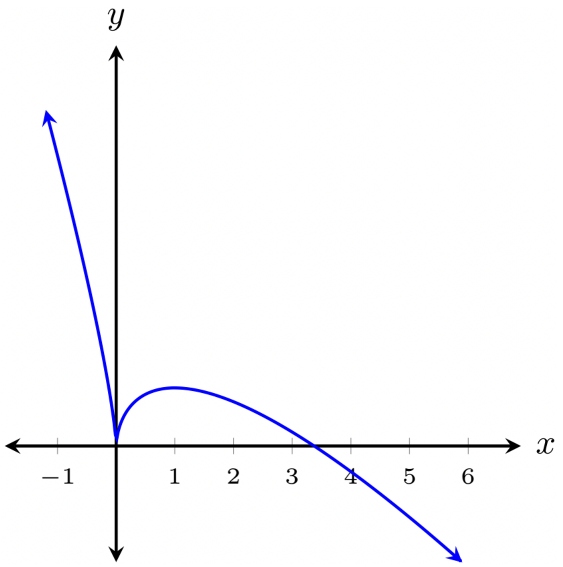
If f is differentiable at $x = a$, then f is continuous at $x = a$.

❗ Converse of this is false. That is, if a function is continuous at a point, then it does not necessarily have to be differentiable.

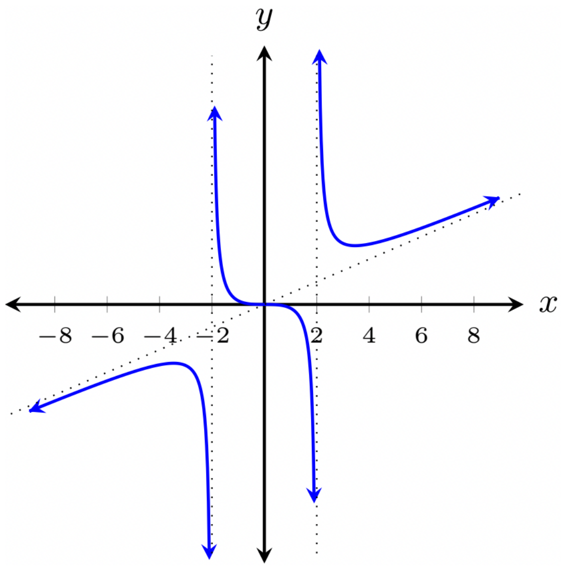
🤔 What kind of continuous functions may not be differentiable everywhere?

Visual Check of continuity and differentiablilty on an interval

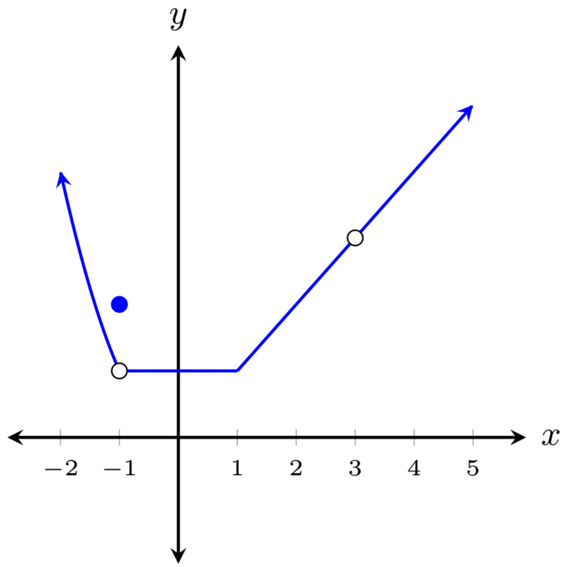
1.



2.



3.



For each of the functions graphed above, identify the points or intervals where the function is

a) Differentiable

b) Continuous but NOT differentiable

c) Neither continuous nor differentiable

Make a sketch of a function meeting all of the following conditions.

$$\lim_{x \to 0^-} f(x) = 2$$

$$\lim_{x \to 0^+} f(x) = -2$$

$$\lim_{x \to 3^-} f(x) = \infty$$

$$\lim_{x \to 3^+} f(x) = 1$$

$f(2) = 0$

$f(0)$ is undefined

$f(x)$ has one point where it is continuous but not differentiable