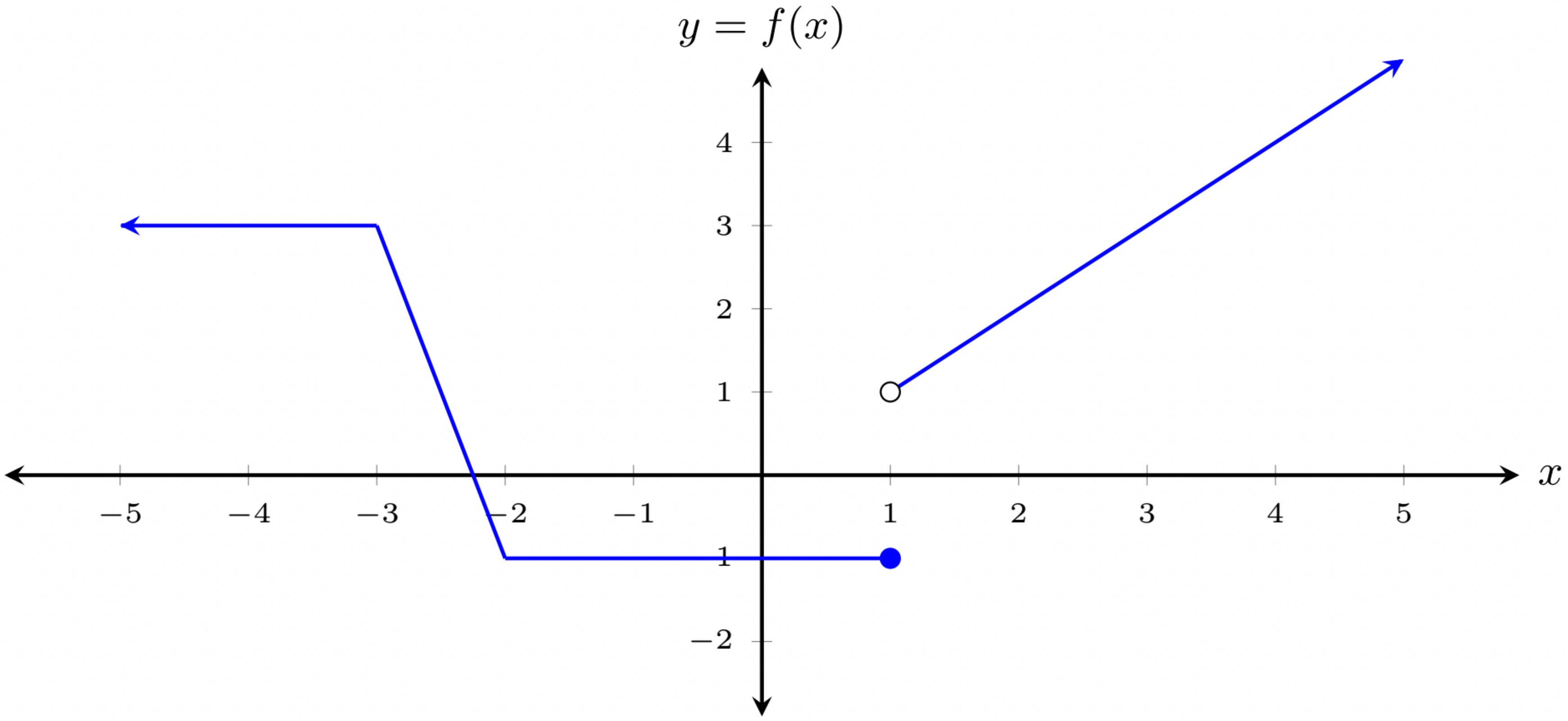
How can we use the alternate form of the derivative to investigate differentiability at a point algebraically?

Quick Check

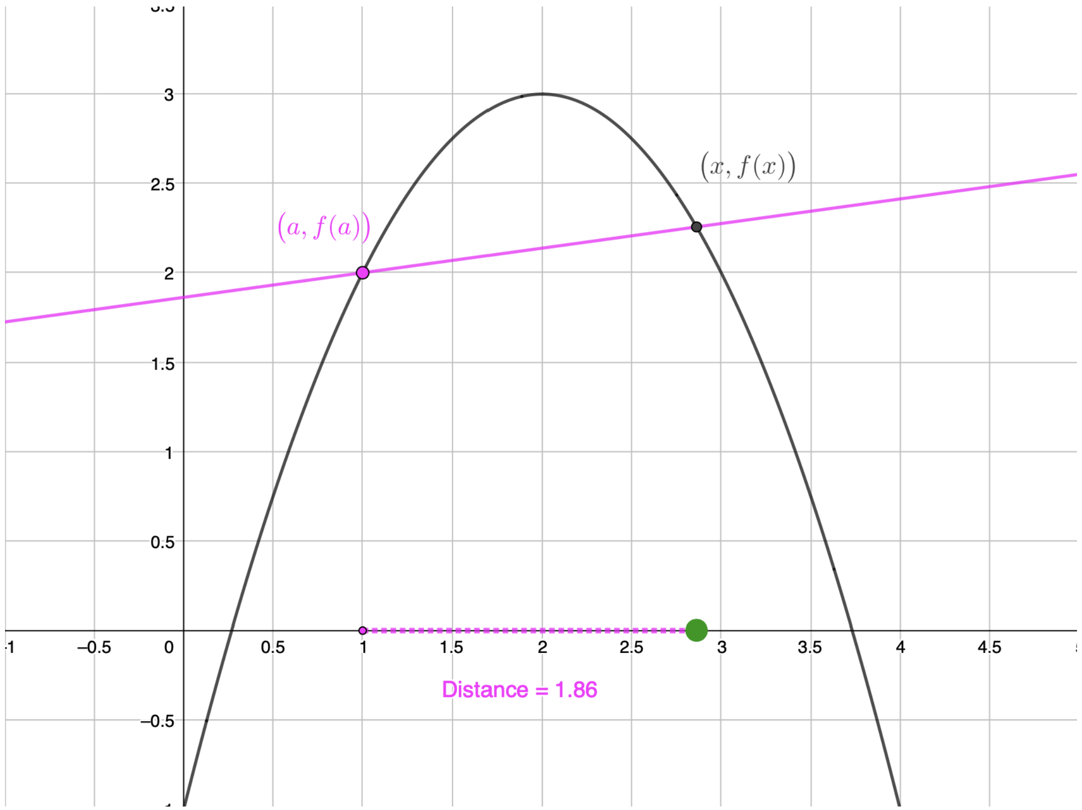
Given $f(x) = \begin{cases} 5 - x & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ x + 2 & \text{if } x > 1 \end{cases}$

Find $\lim\limits_{x \to 1} f(x)$. Show all work.

State, with reasons, the number(s) at which $f(x)$ is not differentiable.



Alternate Form of the Derivative



Geogebra Animation

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The existence of the limit in this alternate form requires that the one-sided limits

$$\lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} \quad \text{and} \quad \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$$

exist and are equal.

Check for differentiability using the alternate form of the derivative

1. $f(x) = (x-6)^{2/3}$

2. $f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq 0 \\ 4 - x^2 & \text{if } x > 0 \end{cases}$

Show Derivative from Left $\neq$ Derivative from Right at the sharp turn

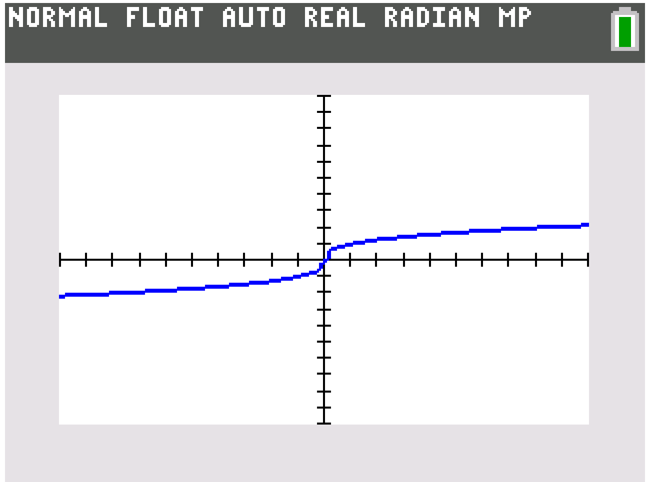
$$f(x) = |x - 4|$$

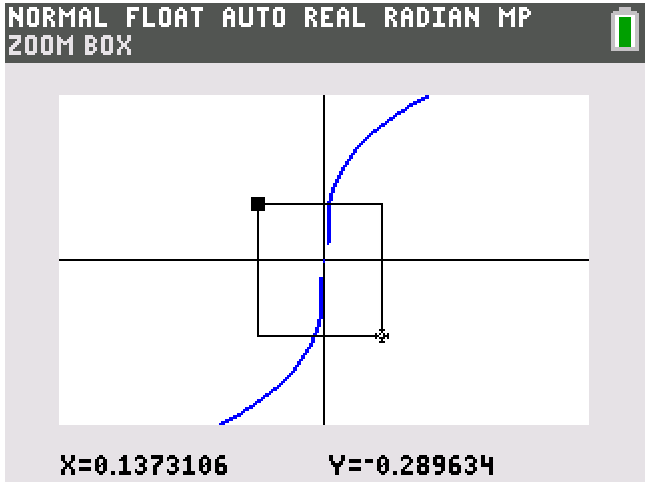
🧐 Start by graphing the function and expressing it as a piece-wise defined function.

Show that the function has a vertical tangent line at $x = 0$

$f(x) = x^{1/3}$

🧐 Start by graphing the function. Think about it algebraically, too.





Differentiability and Piece-wise defined functions

Given $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 2 \\ 4x - 3 & \text{if } x > 2 \end{cases}$

🧐 Start by graphing the piecewise defined function by hand then algebraically check the differentiability at $x = 2$.

🤔 Does having a piece-wise defined function necessarily mean that the function will not be differentiable at the breakpoints of formulas?

Find the derivative using both forms of the derivative side-by-side.

Example: $f(x) = 1 - x^2$, find f'.

$f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$	$f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

1. $f(x) = -7$
2. $g(x) = 4x + 1$
3. $p(x) = \dfrac{1}{x^2}$
4. $f(x) = \dfrac{2}{\sqrt{x}}$