What are the basic differentiation rules?

Quick Check

Use the limit definition of the derivative to find the derivative of each function.

1. $f(x) = c$, where c is a constant.
2. $f(x) = x$
3. $f(x) = x^n$, where n is a positive integer. [✨ Binomial Theorem]

Derivative Rules

1. Derivative of a constant function is 0.

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}[-200] =$$

2. Derivative of the identity function, $f(x) = x$, is 1.

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dt}[t] =$$

3. Derivative of a power function $f(x) = x^n$, where n is a positive integer is $n \cdot x^{n-1}$

$$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$$

$$\frac{d}{dx}[x^8] =$$

Applying the rules

[1-4] Find the derivative using the differentiation rules.

1. $f(x) = 1000$

2. $y = 5\pi$

3. $y = x^2$

4. $g(t) = t^{99}$

5. Find the slope of the graph of $y = x^3$ at $x = -2$, $x = 0$, and $x = 2$.

6. Find an equation of the tangent line to the graph of $f(x) = x^2$ at $(-3, 9)$.

The Constant Multiple Rule

If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$$

🤔 How can we prove this using the limit process?

Differentiate each function:

1. $y = 4x^3$
2. $f(x) = \dfrac{2x^4}{5}$
3. $A(r) = \pi r^2$

The Sum and Difference Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Differentiate each function:

1. $y = x^3 - 4x^2 + 3$
2. $f(t) = \dfrac{t^3 + 2t + 2}{5}$
3. $g(x) = \dfrac{2x^2 - x}{x}$

Apply the derivative rules

1. If $y = x^8 + 12x^5 - 4x^4 + 10x^2 + 6x + 1$, find y'.
2. Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.
3. Show that the graph of $f(x) = x^5 + 3x^3 + 5x$ does not have a tangent line with a slope of 2.

Derivatives of Sine and Cosine Functions

Recall two special trigonometric limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Using these and trig identities we can prove the following two derivatives

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Visual Explanation 🌐 Geogebra Animation

Derivatives of functions involving Sine and Cosine functions

1. $y = x - \sin x$

2. $f(x) = 3x + \cos x$

3. $g(t) = \dfrac{2\sin x}{3}$

4. Find the equation of the tangent line to $\sin x - \cos x$ at the point $(\frac{\pi}{4}, 0)$.

5. Prove $\dfrac{d}{dx}[\cos x] = -\sin x$ using the definition of the derivative.