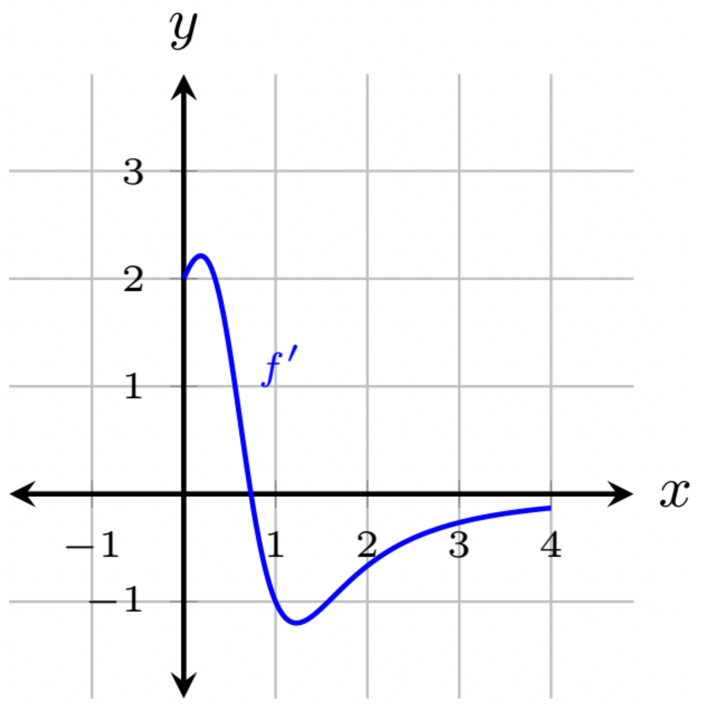
How do we find the derivative of a product of functions?

Quick Check

The function f is defined on the closed interval $[0, 4]$. The graph of its derivative f' is shown. The point $(1, 2)$ is on the graph of $f(x)$.



Write the equation of the tangent line to $y = f(x)$ at $(1, 2)$.

Does the derivative of a product $\frac{d}{dx}[f \cdot g]$ work as expected?

Let $f(x) = x^2$ and $g(x) = 4x + 5$.

1 Find the derivative of $f \cdot g$ by first multiplying the functions then taking the drivative of the result.

2 Check if the product of the derivatives works to achieve the same result.

$$\frac{d}{dx}[f \cdot g] \stackrel{?}{=} \frac{d}{dx}[f] \cdot \frac{d}{dx}[g]$$

✨Simplification Strategy - Addition of Zero ➕0️⃣

Let $p(x) = f(x) \cdot g(x)$, then by the definition of the derivative

$$\begin{aligned} p'(x) &= \lim_{h \to 0} \frac{p(x+h) - p(x)}{h} \\ &= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \dots \end{aligned}$$

The Product Rule for Differentiation

The product of two differentiable functions is itself differentiable.

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$
$$= 1^{st} \text{ function} \cdot \text{derivative } 2^{nd} + 2^{nd} \text{ function} \cdot \text{derivative } 1^{st}$$

$$1^{st} \cdot 2^{nd}$$
$$f(x) = x^3 \cdot \sin x$$

$$f'(x) =$$

Use the product rule to differentiate each function.

1. $f(x) = (2x + 3)(x^3 - 4)$

2. $g(t) = t^3 \cdot (2 - t^2)$

3. $g(x) = \sin x \cdot \cos x$

4. $p(x) = f(x) \cdot g(x) \cdot h(x)$ $\left[\text{Hint✨}\ p(x) = (f \cdot g) \cdot h\right]$

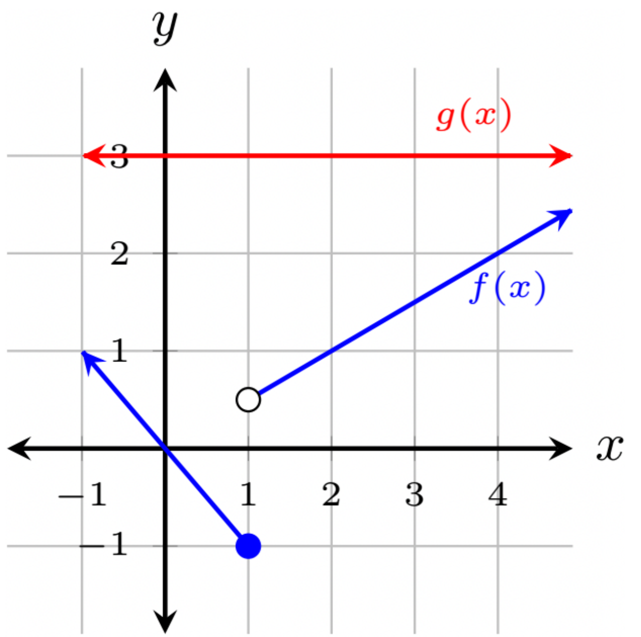
5. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

More Derivatives

1. Find the derivative of $f(x) = (2 + x^2)(x - x^2)$ in two ways side by side using the product rule and by multiplying first. Are the results the same?

2. If $f(2) = 10$ and $f'(x) = x^2 f(x)$ for all x, find $f''(2)$.

3. f and g are the functions whose graphs are shown to the right. Let $p(x) = f(x)g(x)$. Find $p'(2)$.



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