How can we find the derivative of a composite function $(f \circ g)$?

Quick Check

Determine whether the statement is true or false. If it is false, explain why or give an example to show that it is false.

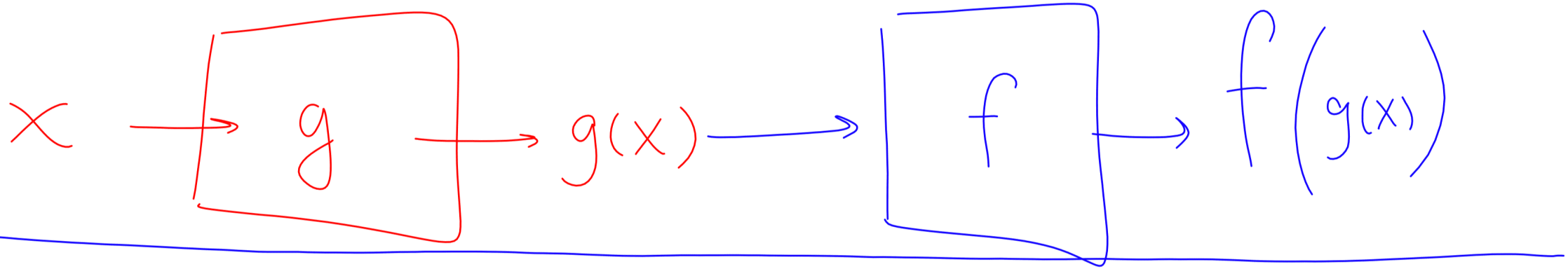
1. If $y = f(x)g(x)$, then $\dfrac{dy}{dx} = f'(x)g'(x)$.

2. If $f(x)$ is an n^{th} degree polynomial, then $f^{(n+1)}(x) = 0$.

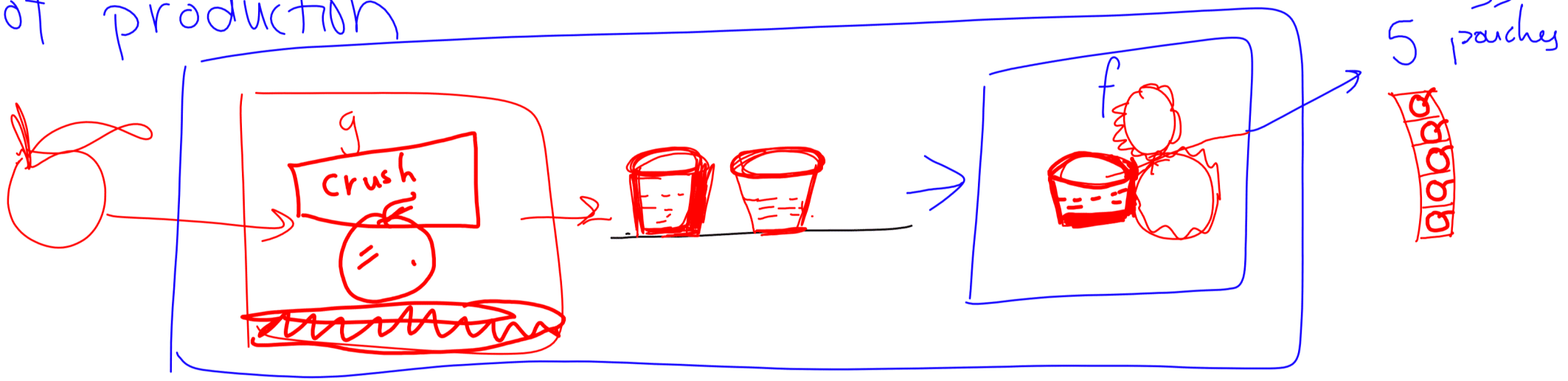
3. If a function is continuous at a point, then it is differentiable at that point.

Factory Operation and Production

$(f \circ g)(x) = f(g(x))$



rate of production



The Chain Rule

If f is differentiable at $g(x)$ and g is differentiable at x, then the composite function $(f \circ g)(x)$ is differentiable at x, and

$$(f \circ g)'(x) = f'\Big(g(x)\Big) \cdot g'(x)$$
$$= \text{derivative of outside (leave inside alone)} \cdot \text{ derivative of inside}$$

Find the derivative.

1. $y = (5x - 2)^6$
2. $f(x) = (3x - 2x^2)^3$

More Examples of derivatives using the Chain Rule

Differentiate

1. $g(t) = \dfrac{10}{(2t-4)^3}$

2. $f(x) = x^2 \cdot \sqrt{1-x^2}$

3. $f(x) = \left(\dfrac{2x-1}{x^2+1}\right)^2$

4. $y = (x^2-2)^3(2x^4+2)^2$

Find the derivative of each function.

1. $y = (6x - 5)^3$
2. $y = 5(1 - x^2)^{-4}$
3. $f(x) = x^2(x - 2)^4$
4. $g(t) = \dfrac{2}{(t^4 + 1)^3}$
5. $h(x) = \left(\dfrac{x^2 - 2}{2x + 3}\right)^2$
6. Find the point(s) on the curve of $f(x) = \sqrt[3]{(x^2 - 1)^2}$ where the slope is zero or undefined.