How do we find the derivatives of trigonometric functions using the Chain Rule?

Quick Check

Determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

If $f(x) = (x^6 - x^4)^5$, then $f^{(31)}(x) = 0$.

Trigonometric Functions and the Chain Rule

Let $u(x)$ be a differentiable function.

$$\frac{d}{dx}[\sin u] = (\cos u) \cdot u' \qquad \frac{d}{dx}[\cos u] = -(\sin u) \cdot u' \qquad \frac{d}{dx}[\tan u] = (\sec^2 u) \cdot u'$$

$$\frac{d}{dx}[\csc u] = -(\csc u \cot u) \cdot u' \qquad \frac{d}{dx}[\sec u] = (\sec u \tan u) \cdot u' \qquad \frac{d}{dx}[\cot u] = -(\csc^2 u) \cdot u'$$

Differentiate: 1 $y = \sin(x^2 + 9)$ 2 $y = \tan(2x)$ 3 $y = x^2 \cdot \sin(4x)$

⚠️ Careful with notation

Find the derivative of each function.

1. $y = \cos 3x^2$	2. $y = (\sin 3)x^2$	3. $y = \sin(4x)^2$	4. $y = \sin^2 x$	5. $y = \sqrt{\sin x}$

Repeated use of the Chain Rule

Find the derivative of the function.

1. $y = \cos^2(5x)$
2. $y = \sin(x^2 + \sin 2x)$ [Visualize y and y' on Desmos or a graphing calculator]

Practice

1. Find the equation of the tangent line to $y = tan(2x)$ at $x = \pi$.

2. Find the second derivative of $y = x \cos x$.

3. Find the derivative of $y = \sin^2 x$ in two ways.

4. Find the derivative of $y = \sin(\pi x)^2$.

5. Given $y = x^3 \sin^2(4x)$, find $\dfrac{dy}{dx}$.

6. Find the values of $x \in [0, 2\pi]$ where $y = \sin x \cdot \cos x$ has a horizontal tangent line.