How can we find derivatives of implictly defined functions?

Quick Check

1. Find the point(s) on the graph of the equation $x^2 + y^2 = 25$ whose x-coordinate is 4.

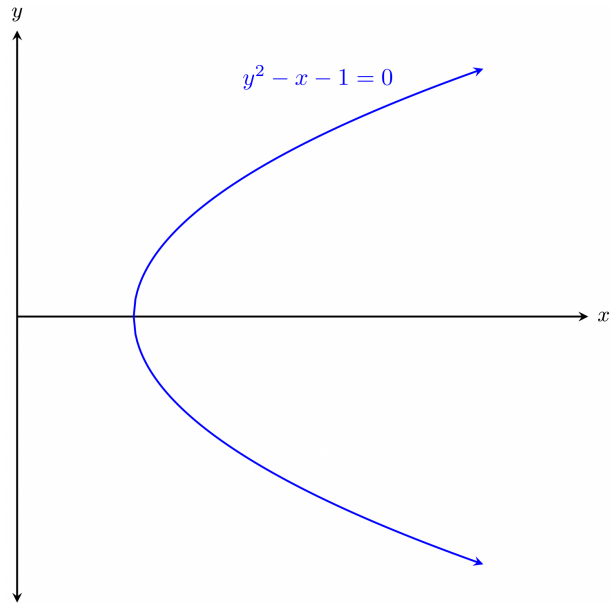

2. How many points have the x-coordinate equals zero, $x = 0$, on the graph of the equation $y^4 = y^2 - x^2$.

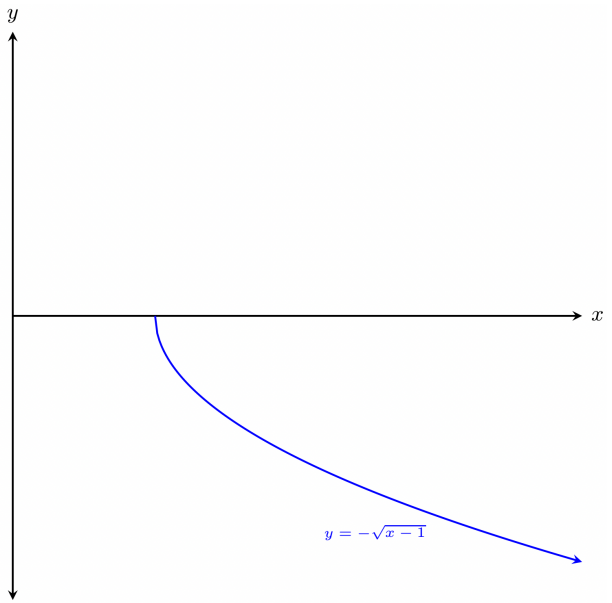

3. Find the slope of the tangent line to the graph of the function $f(x) = (2x^3 + 1)^2$ at $x = -1$.

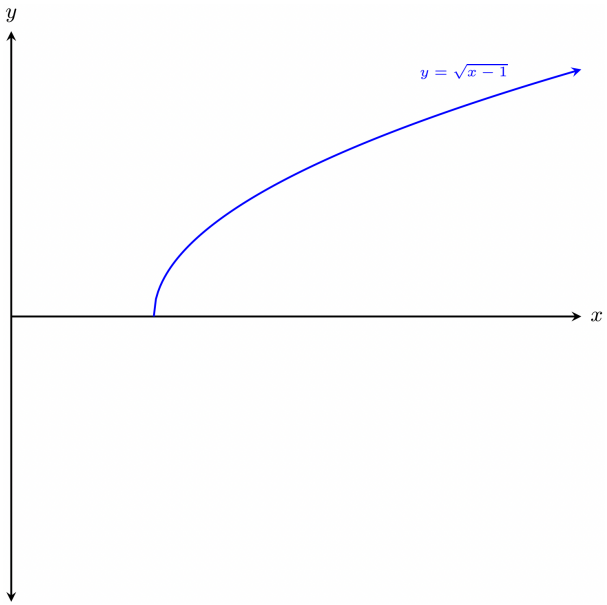
Implicitly vs Explicitly defined functions

Explicit Form	Implicit Form
$y = 1/x$	$xy = 1$
$y = \sqrt{1 - x^2}$ and $y = -\sqrt{1 - x^2}$	$x^2 + y^2 = 1$
😅 Solve for y	$x^3 + y^3 = 6xy$

Implicitly vs Explicitly defined functions







$y^2 - x + 1 = 0$ defines $y = \sqrt{x-1}$ and $y = -\sqrt{x-1}$ implicitly.

$[f(x)]^2 - x + 1 = 0$ defines $f(x) = \sqrt{x-1}$ and $f(x) = -\sqrt{x-1}$ implicitly.

Implicit Differentiation

$$y = 1/x$$

$$xy - 1 = 0$$

Implicit Differentiation

$f(x) = \sqrt{x-1}$ and $f(x) = -\sqrt{x-1}$

$[f(x)]^2 - x + 1 = 0$

Implicit Differentiation Practice

Find the derivative using implicit differentiation method.

Examples

1. $x^3 - xy + y^2 = 4$
2. $y^3 + y^2 - 5y - x^2 = -4$
3. Find $\dfrac{d^2y}{dx^2}$ if $y^2 = x^3$
4. $5y^2 + \sin y = x^2$
5. $\sin x + 2\cos(2y) = 1$
6. Find $\dfrac{d^2y}{dx^2}$ if $4x^2 - 2y^2 = 9$
7. Find the slope of the curve $y^4 = y^2 - x^2$ at $(0, 1)$. Check by graphing.

Orthogonal or Normal Lines

Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$. Sketch the graph along with tangent and normal line through the given point. Does the sketch verify your algebraic solution?

General Power Rule - fractional exponents

$$y = x^{p/q} \quad \longrightarrow \quad y^q = x^p$$