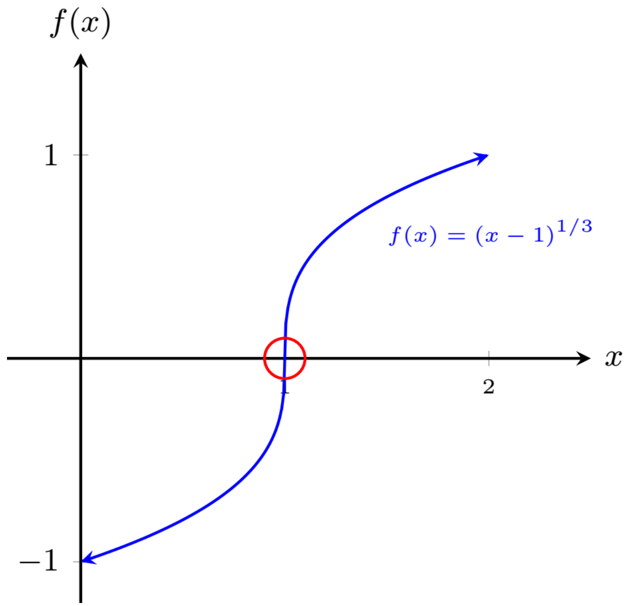
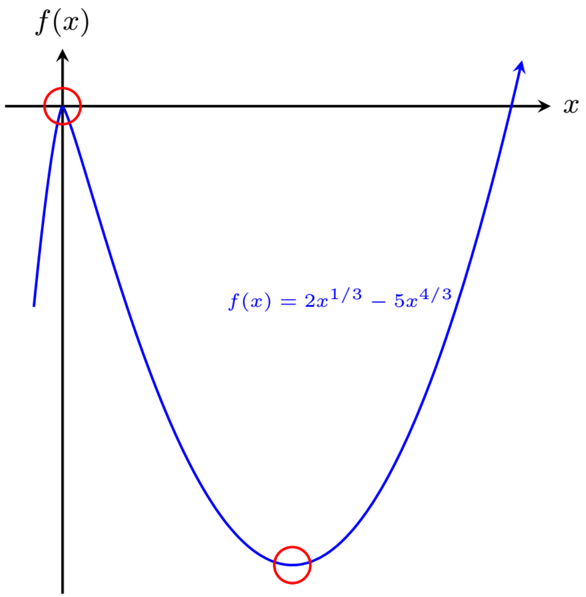
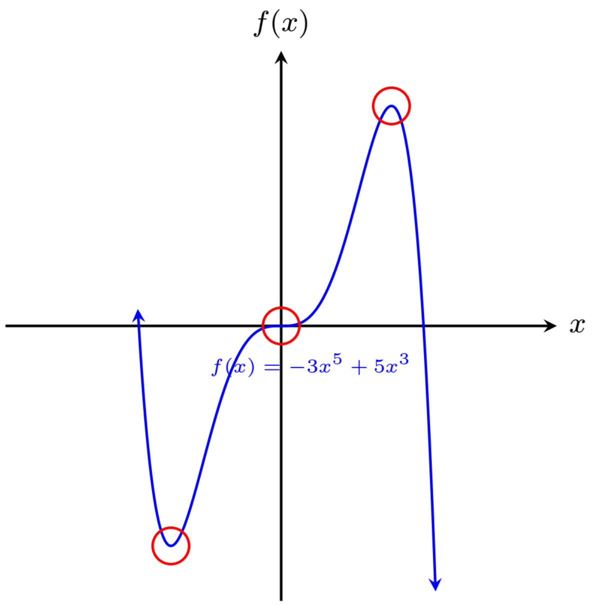
What does the Extreme Value Theorem say?

Quick Check

Visually find the derivatives of the graphed functions at the indicated points.







Extreme Values

The minimum and maximum values of a function are called the extreme values.

1️⃣ We say $f(x)$ has an absolute (or global) maximum at $x = c$ if $f(x) \leq f(c)$ for every x in the domain we are working on.

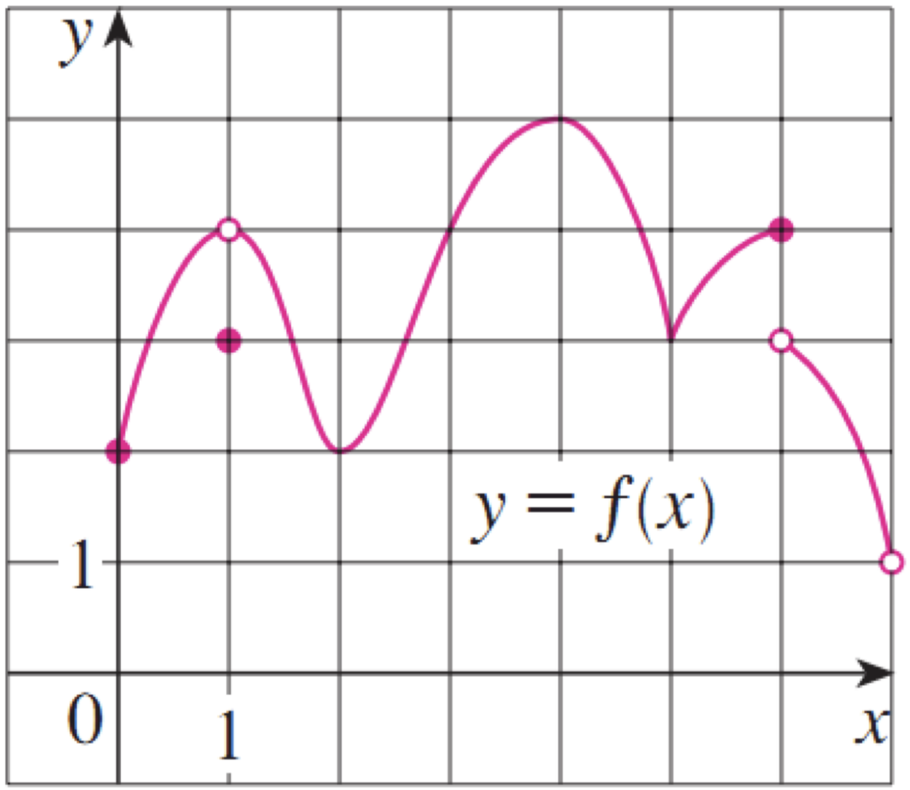
2️⃣ We say $f(x)$ has an absolute (or global) minimum at $x = c$ if $f(x) \geq f(c)$ for every x in the domain we are working on.

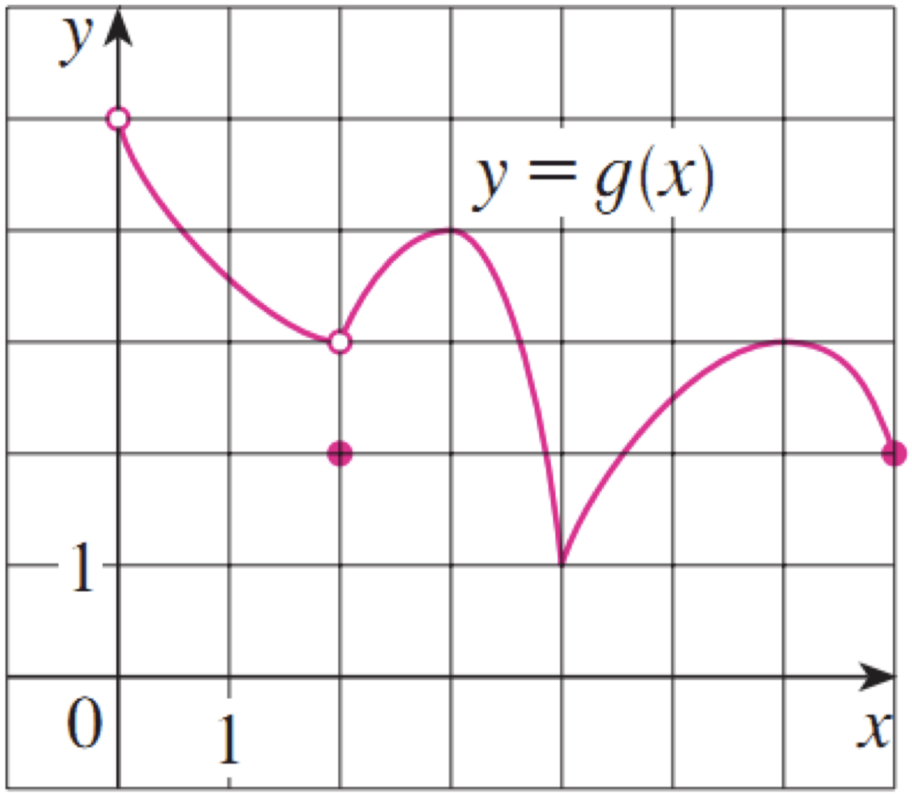
3️⃣ We say $f(x)$ has an relative (or local) maximum at $x = c$ if $f(x) \leq f(c)$ for every x in some open interval around $x = c$.

4️⃣ We say $f(x)$ has an relative (or local) minimum at $x = c$ if $f(x) \geq f(c)$ for every x in some open interval around $x = c$.

Practice Reading Graphs

Decide whether each $x \in \{1, 2, 3, 4, 5, 6, 7\}$ is an absolute maximum or minimum, a relative maximum or minimum, or neither.





Practice Sketching

1️⃣ Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties.

- Absolute minimum at 2, absolute maximum at 3, local minimum at 4
- Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4

2️⃣ Using your 🖥️, examine the graph of $f(x) = 2x - 3x^{2/3}$ for any relative extrema on $[-1, 3]$. Find the derivative at each extreme point(s) algebraically.

Critical Number

A number c in the domain of the function f is called a critical number if either $f'(c) = 0$ or $f'(c)$ does not exist.

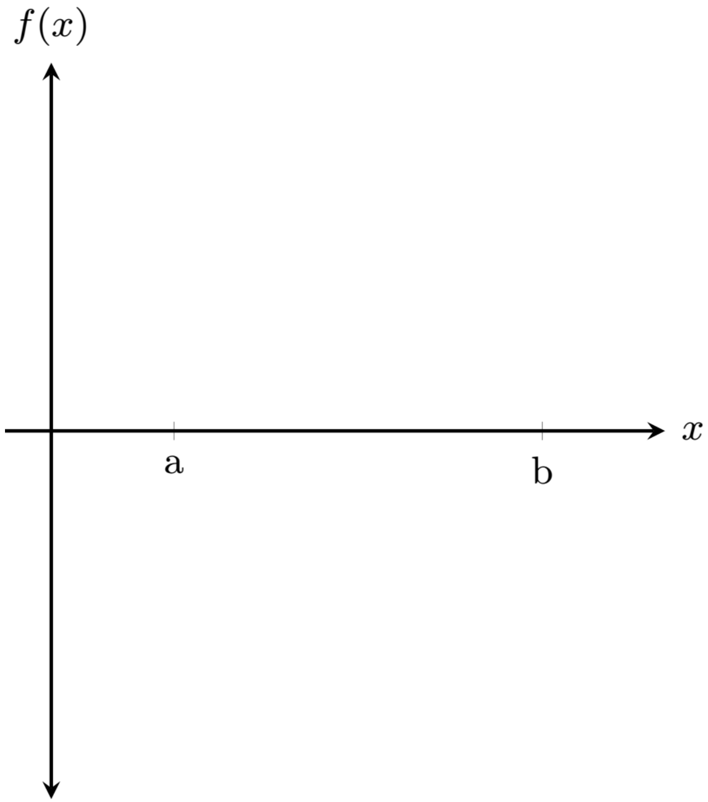
⭐ Critical numbers provide candidates for relative extrema.

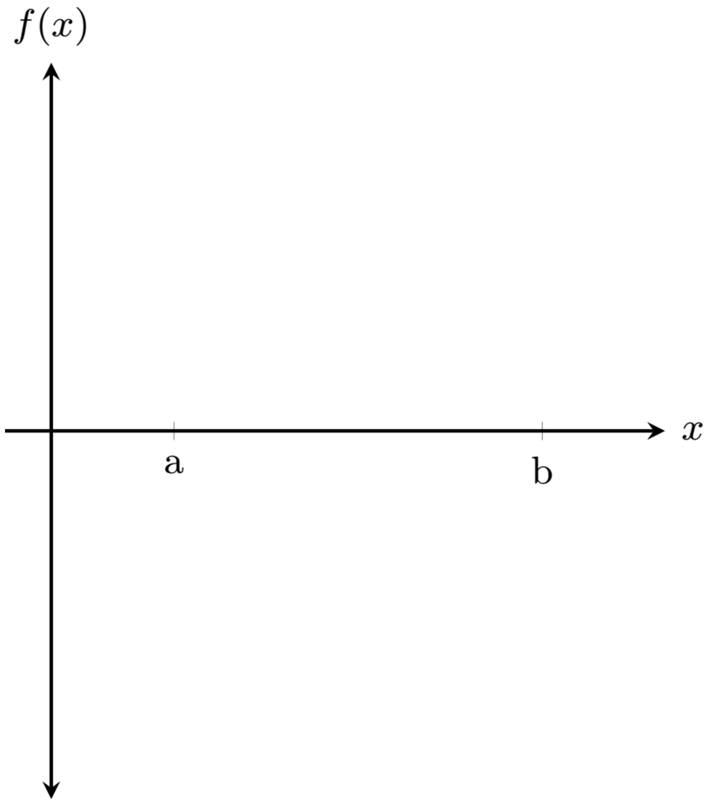
- Relative extrema at $x = c \implies$ critical number at $x = c$
- However, critical number at $x = c \not\Longrightarrow$ Relative extrema at $x = c$

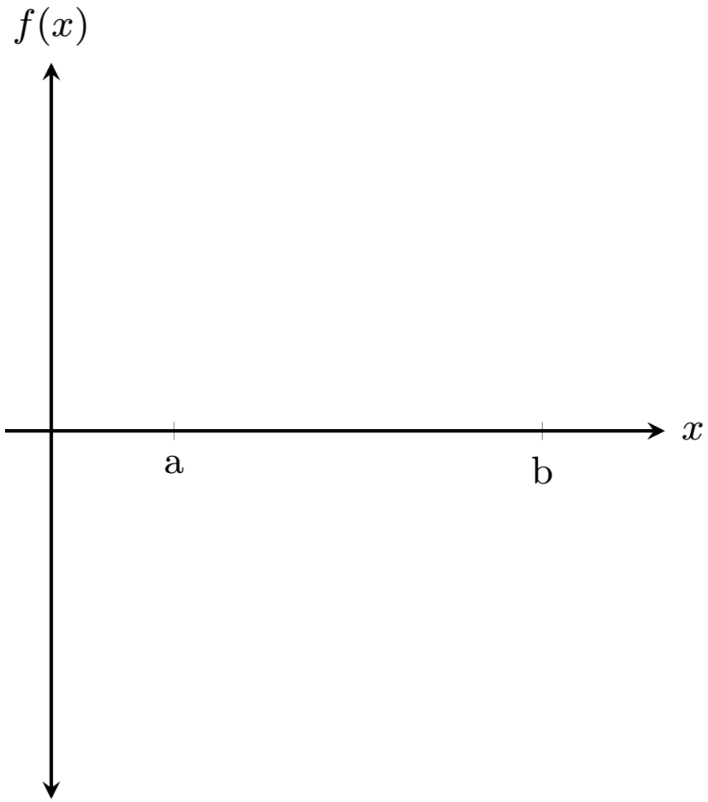
Let's go back and review our work in the lesson so far to spot critical numbers.

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value and an absolute minimum value at some numbers in the closed interval.







Applying EVT

1. Find the extrema of $f(x) = \dfrac{x^2}{x^2+3}$ on $[-1, 1]$.

2. Find the extrema of $f(x) = 3x^4 - 4x^2$ on $[-1, 2]$.

3. Find the extrema of $f(x) = cos(\pi x)$ on $[0, \dfrac{1}{6}]$.