In what ways can we use the conclusions of the existence theorem known as the Mean Value Theorem?

Quick Check (Multiple Choice)

If f is a continuous function on $[a, b]$, which of the following is necessarily true?

a. f' exists on (a, b)

b. If $f(x_0)$ is a maximum of f, the $f'(x_0) = 0$

c. $\lim_{x \to x_0} f(x) = f(\lim_{x \to x_0} x)$ for $x_0 \in (a, b)$

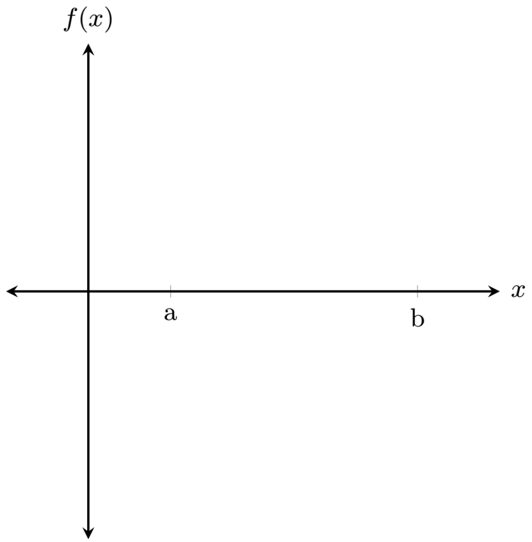
d. $f'(x) = 0$ for some $x \in [a, b]$

e. The graph of f' is a straight line

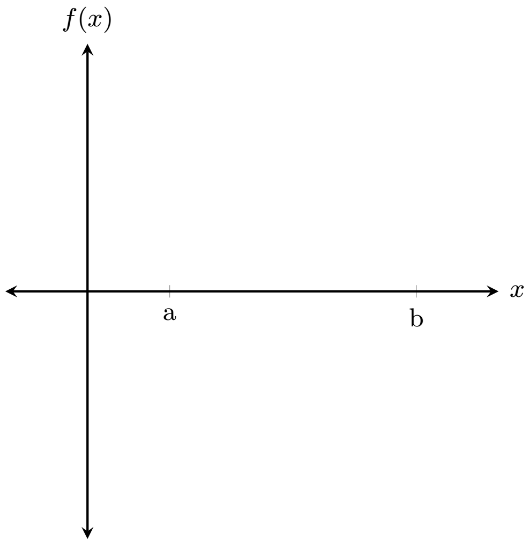
Give full reasoning for selecting or rejecting any of the multiple choices.

Draw a sketch of a function meeting the set requirements

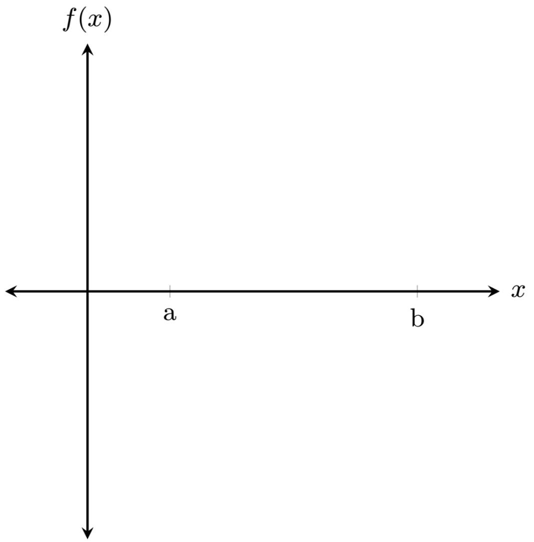
- f is NOT continuous on the closed interval $[a, b]$
- f is differentiable on the open interval (a, b)
- $f(a) = f(b)$



- f is continuous on the closed interval $[a, b]$
- f is NOT differentiable on the open interval (a, b)
- $f(a) = f(b)$



- f is continuous on the closed interval $[a, b]$
- f is differentiable on the open interval (a, b)
- $f(a) \neq f(b)$



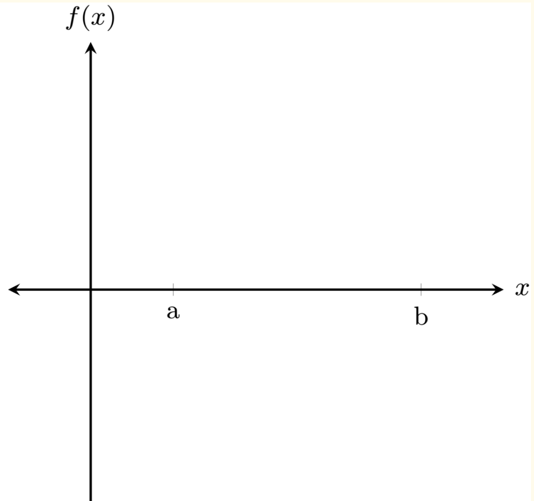
Rolle's Theorem

Let f be a function that satisfies the following three hypothesis:

- f is continuous on the closed interval $[a, b]$
- f is differentiable on the open interval (a, b)
- $f(a) = f(b)$

Then there is a number c in the open interval (a, b) such that $f'(c) = 0$

⭐ See your drawings from last slide for the necessity of each of the conditions. Now, draw a graph meeting all three conditions and check if the conclusion follows.



Existence of a solution

1️⃣ Find two $x-$intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.

2️⃣ Let $f(x) = x^4 - 2x^2$. Find all values of c in the interval $(-2, 2)$ such that $f'(c) = 0$.

3️⃣ Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this NOT contradict Rolle's Theorem?

If DMV used map data and did the math 😱

You are driving on a straight highway on which the speed limit is $55 mi/h$. At 8:05 AM, a police car clocks your velocity at $50 mi/h$ and at 8:10AM, a second police car posted $5 mi$ down the road clocks your velocity at $55 mi/h$. Explain why the police have a right to charge you with a speeding violation.

Mean Value Theorem (MVT)

Let f be a function that satisfies the following hypothesis:

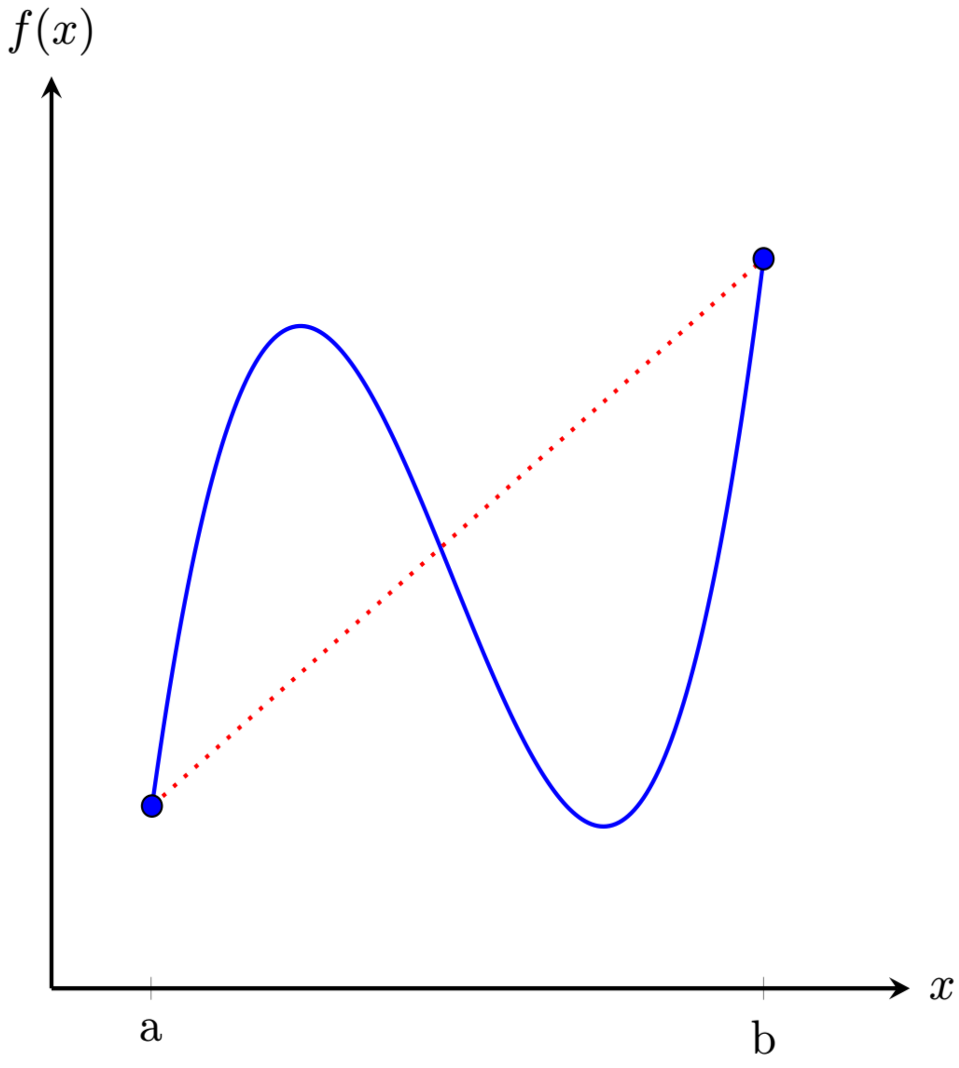
- f is continuous on the closed interval $[a, b]$
- f is differentiable on the open interval (a, b)

Then there is a number c in the open interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\text{Instantaneous Rate of Change} = \text{Average Rate of Change}$$

Geometric View of MVT



- ☐ f is continuous on the closed interval $[a, b]$
- ☐ f is differentiable on the open interval (a, b)

$$\implies c \in (a, b) \text{ such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$

Where ?

Practice

1️⃣ Verify that the function satisfies the hypothesis of the Mean Value Theorem on the given interval. Then find all numbers that satisfy the conclusion of the MVT.

a. $f(x) = 3x^2 + 2x + 5,\ [-1, 1]$

b. $f(x) = \dfrac{x}{x+2},\ [1, 4]$

2️⃣ Why is the Rolle's Theorem a special case of the Mean Value Theorem?