What does f'' say about f ?

Quick Check

Recall the previous lesson. Summarize what f' says about f.

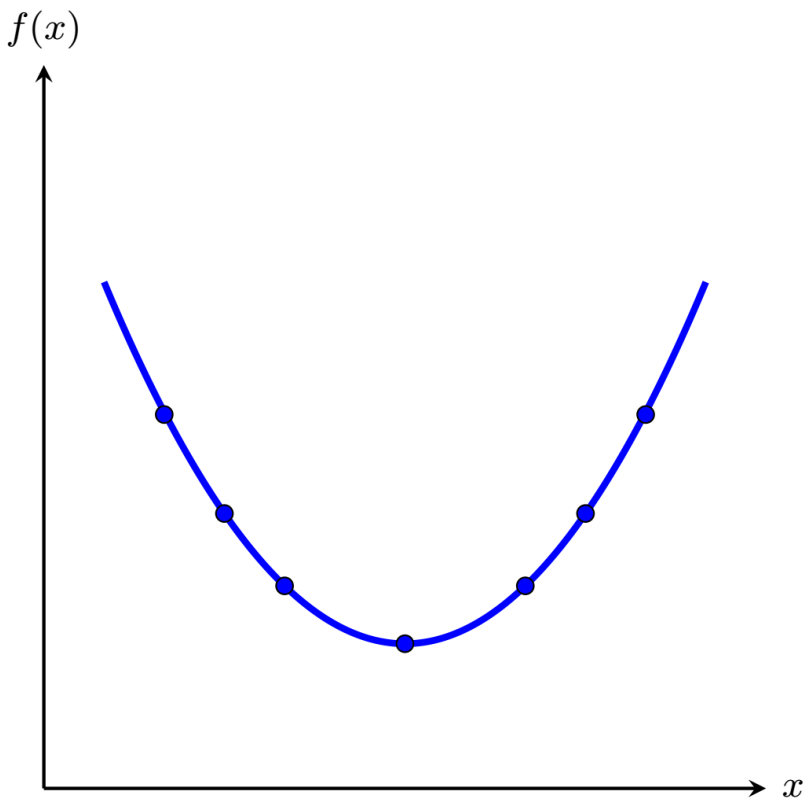
f'	f
$+$	
$-$	
0	

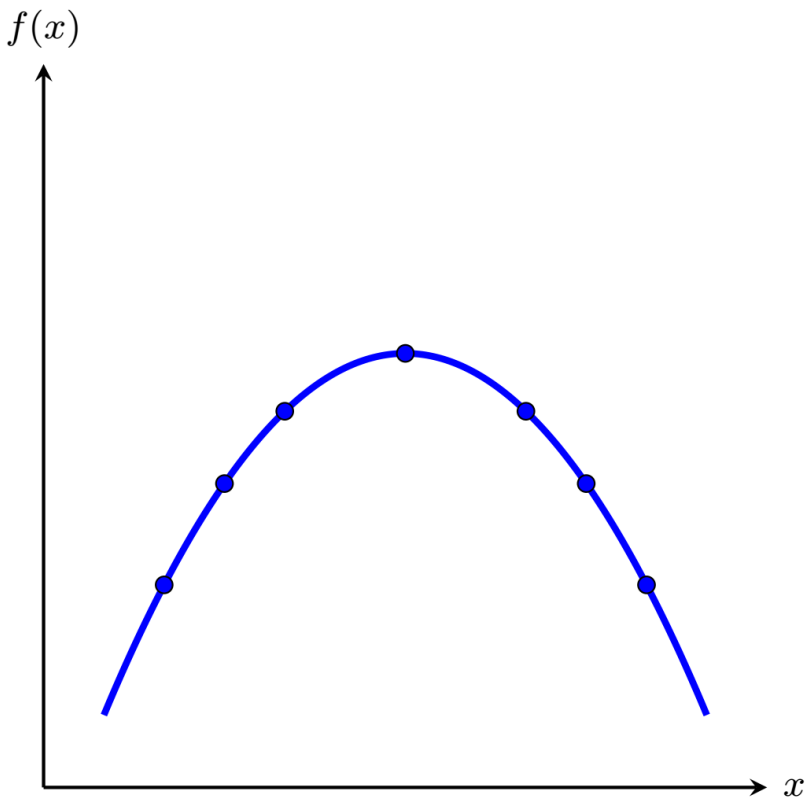
Now think about what f'' would say about f' ...

f''	f'	...
$+$		
$-$		
0		

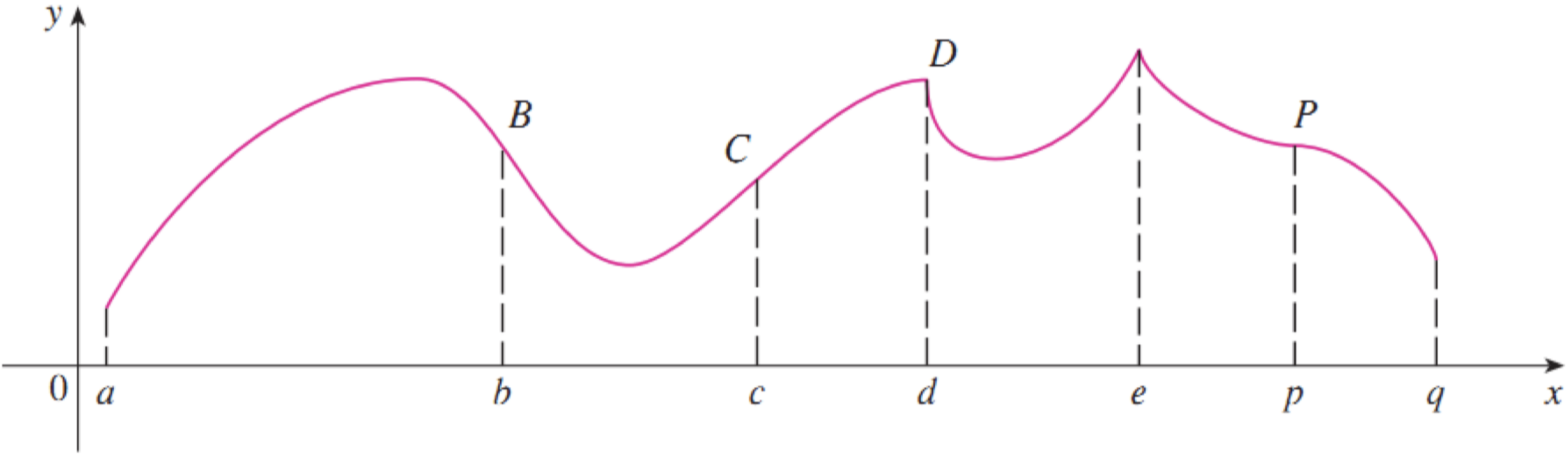
Concavity

Draw a tangent segment at each highlighted point. Assign a numerical estimate for the slope to each. What do you notice the shape of the function and change in slopes?





🔍 Concave Up / Concave Down / Point of Twist (inflection)

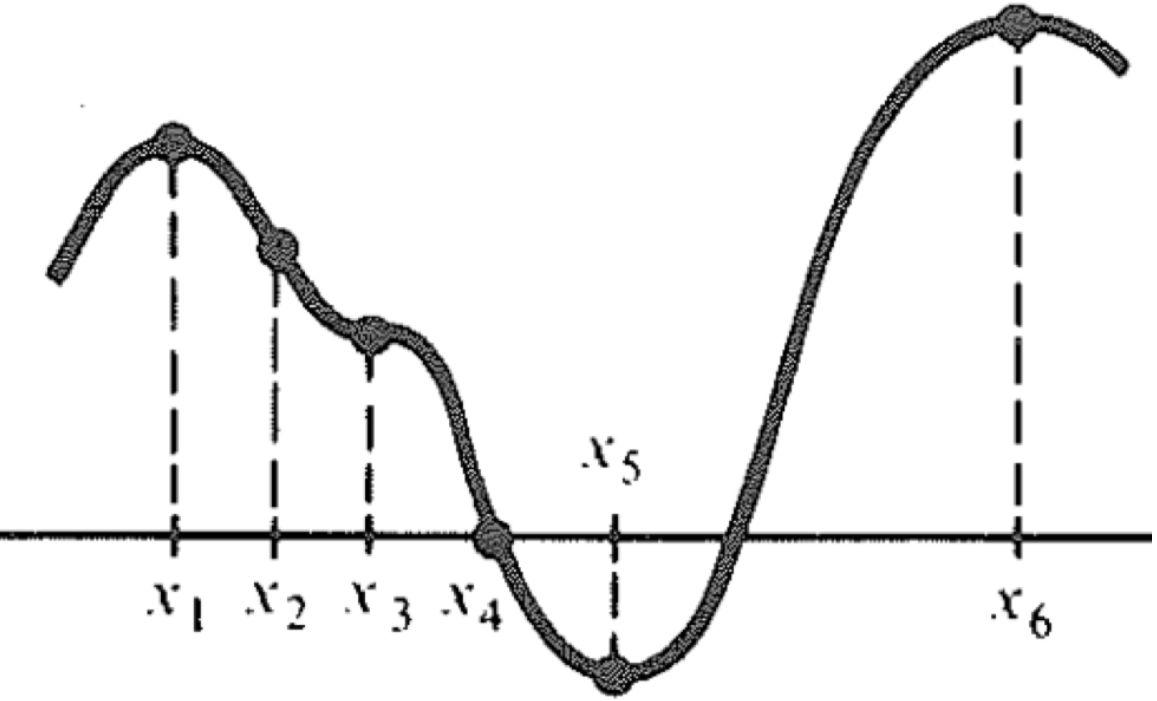


Determining Concavity

Locate candidates for point of inflection by investigating where second derivative of f is zero or undefined. ⚠️ f must be defined at this point.

1️⃣ Determine the point(s) of inflection and discuss the concavity of the graph of $f(x) = x^3 - 6x^2 + 12x$.

2️⃣ Determine the point(s) of inflection and discuss the concavity of the graph of $f(x) = x^4 - 4x^3$.



How could we locate extrema using the concavity of a function?

At the critical number c of f where $f'(c) = 0$

- If $f''(c) > 0 \implies f$ is concave up and there is a rel min at $\big(c, f(c)\big)$
- If $f''(c) < 0 \implies f$ is concave down and there is a rel max at $\big(c, f(c)\big)$
- If $f''(c) = 0 \implies$ use another method (1st Derivative Test)

Apply the Second Derivative Test

1. Find the relative extrema for $f(x) = -3x^5 + 5x^3$.

2. Find all relative extrema for $f(x) = x^4 - 4x^3 + 2$.

3. Find all relative extrema for $f(x) = x^2(6 - x)^3$.

📌 Both 1st Derivative Test and 2nd Derivative Test are used to locate extrema. 1st uses the idea of increase/decrease and 2nd tries to use the idea of concavity to achieve the same.