How do we determine the horizontal asymptotes of a function ?

Quick Check

1 Find the vertical asymptotes, if any, of the graph of the function.

$$h(x) = \frac{4x}{4 - x^2}$$

2 What is a vertical asymptote (in your own words)?

Recall Vertical Asymptotes and Limits

1. $\lim_{x \to -2^-} f(x)$

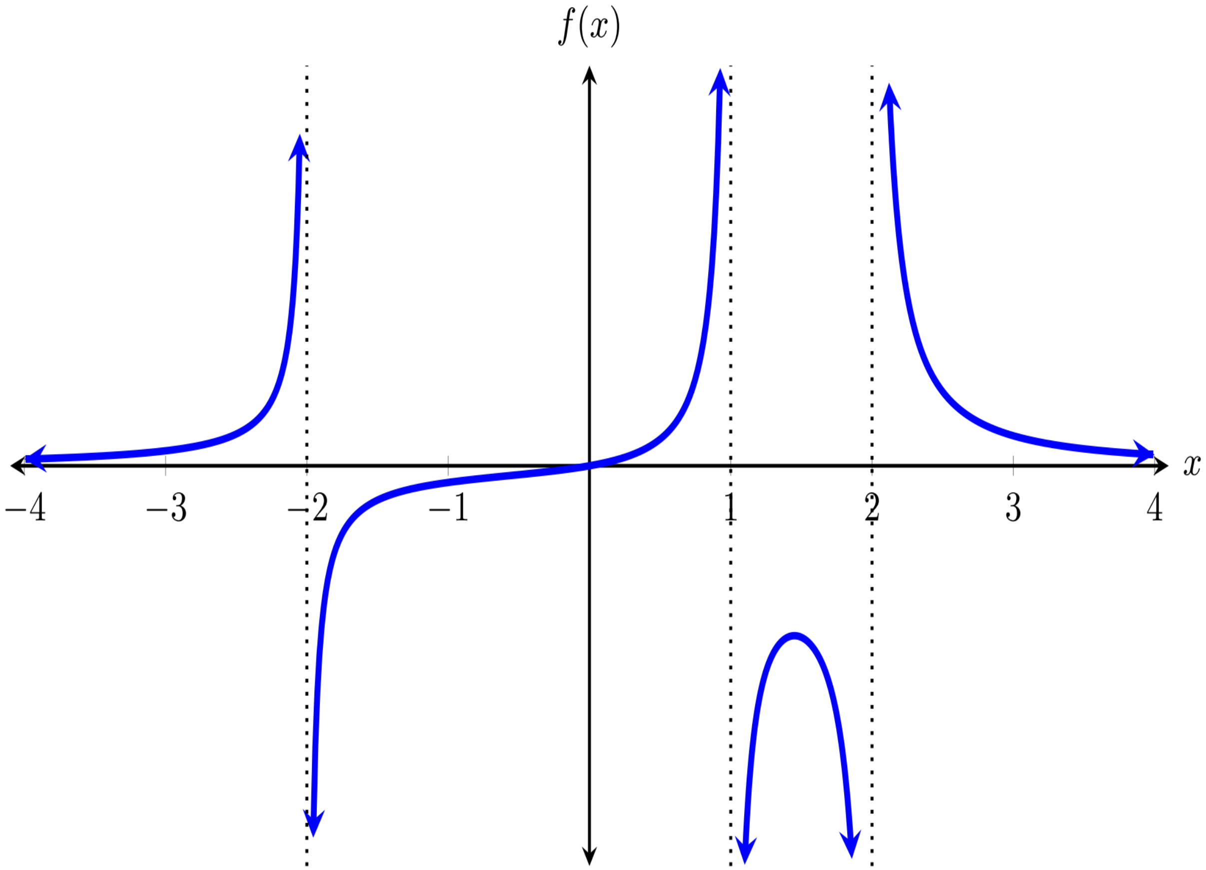
2. $\lim_{x \to -2^+} f(x)$

3. $\lim_{x \to 1} f(x)$

4. $\lim_{x \to 2^-} f(x)$

5. $\lim_{x \to 2^+} f(x)$

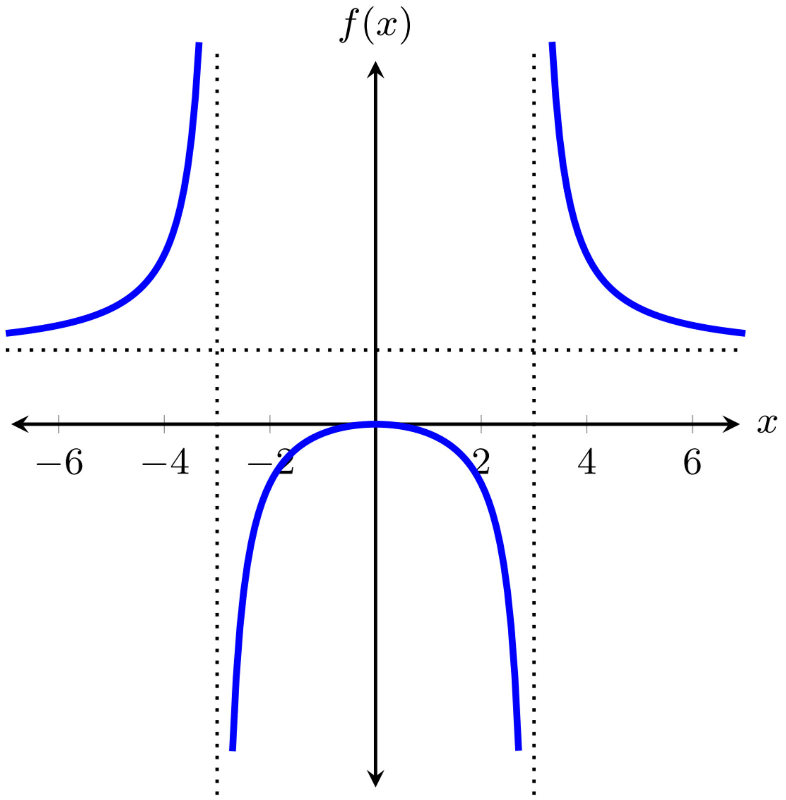
6. Equations of vertical asymptotes are?

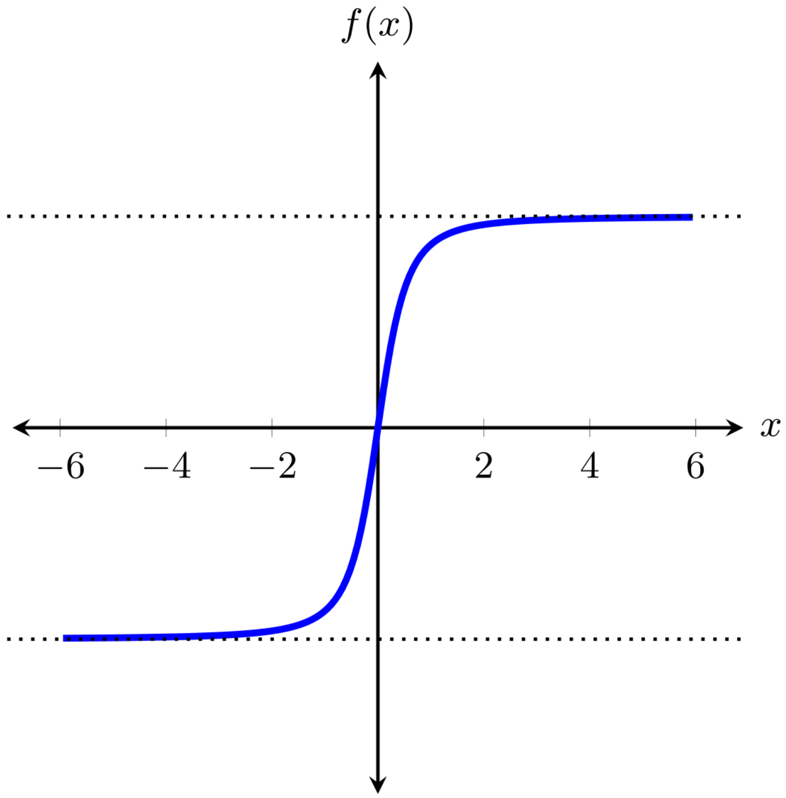


Horizontal Asymptote

A horizontal line $y = L$ is a horizontal asymptote for a function f if

$$\lim_{x \to \pm\infty} f(x) = L$$





Guidelines for finding limits of rational functions

1. $\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$

2. $\lim_{x \to \infty} \frac{2x - 1}{x^5 + 1}$

3. $\lim_{x \to \infty} \frac{2x^3 + 5}{3x^2 + 1}$

❓ What about $\lim_{x \to -\infty} f(x)$?

1. Degree of numerator < degree of denominator
   $\implies$ limit of rational function $= 0$.

2. Degree of numerator = degree of denominator
   $\implies$ limit of rational function $=$ ratio of leading coefficients.

3. Degree of numerator > degree of denominator
   $\implies$ limit of rational function DOES NOT EXIST.

What does the function do at the extreme x values?

$$f(x) = \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

Check limits at ∞ and $-\infty$

1. $$\lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

$$\lim_{x \to -\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

2. $f(x) = \frac{3x^2}{x^2 + 2}$
3. $f(x) = \frac{4\, sin(x)}{x^2 + 1}$
4. $f(x) = \frac{2x}{\sqrt{x^2 + 2}}$
5. $f(x) = \frac{2x^2 - 3x + 5}{x^2 + 1}$

Sketch the graph of a function that satisfies all of the given conditions.

1	2
$f'(0) = f'(2) = f'(4) = 0$	$f'(1) = f'(-1) = 0$
$f'(x) > 0$ if $x < 0$ or $2 < x < 4$	$f'(x) < 0$ if $-1 < x < 1$
$f'(x) < 0$ if $0 < x < 2$ or $x > 4$	$f'(x) > 0$ if $1 < x < 2$ or $-2 < x < -1$
$f''(x) > 0$ if $1 < x < 3$	$f''(x) < 0$ if $-2 < x < 0$
$f''(x) < 0$ if $x < 1$ or $x > 3$	inflection point $(0, 1)$

Sketch the graph of a function that satisfies all of the given conditions.

3

$$\lim_{x\to 2} f(x) = -\infty$$

$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = 0$$

$$\lim_{x\to 0^+} f(x) = \infty$$

$$\lim_{x\to 0^-} f(x) = -\infty$$

4

$$f(0) = 4$$

$$\lim_{x\to 0^-} f(x) = 4$$

$$\lim_{x\to 0^+} f(x) = 2$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

$$\lim_{x\to 4^-} f(x) = -\infty$$

$$\lim_{x\to 4^+} f(x) = \infty$$

$$\lim_{x\to\infty} f(x) = 3$$