How can we draw a reliable graph of a function without a calculator?

## Quick Check

Find the horizontal and vertical asymptotes of the following function, if any.

$$
f(x)=\frac{x}{\sqrt{x^{2}-9}}
$$

## Sketching - Visual Practice

1 Handout - Data to Graph

| Domain | ( $-\infty, \infty$ ) |
| :---: | :---: |
| Intercepts | $y$-intercept: 1 |
| Symmetry | None |
| Asymptotes | None |
| Intervals where $f$ is $\nearrow$ or $\backslash$ | $\begin{aligned} & \text { on }(-\infty, 0) \text { and on }(2, \infty) ; \\ & \text { on }(0,2) \end{aligned}$ |
| Relative extrema | Rel. max. at ( 0,1 ); rel. min. at $(2,-3)$ |
| Concavity | Downward on ( $-\infty, 1$ ); upward on $(1, \infty)$ |
| Point of inflection | $(1,-1)$ |
| Domain | $(-\infty, 0) \cup(0, \infty)$ |
| Intercepts | $x$-intercept: 1 |
| Symmetry | None |
| Asymptotes | $x$-axis; $y$-axis |
| Intervals where $f$ is $\boldsymbol{\sim}$ or $\backslash$ | $\begin{aligned} & \nearrow \text { on }(0,2) ; \backslash \text { on }(-\infty, 0) \\ & \text { and on }(2, \infty) \end{aligned}$ |
| Relative extrema | Rel. max. at ( 2,1 ) |
| Concavity | Downward on $(-\infty, 0)$ and on $(0,3)$; upward on $(3, \infty)$ |
| Point of inflection | $\left(3, \frac{8}{9}\right)$ |

2 Handout + Desmos - Graph to Data


## Sketching - Algebraic Practice

Sketch the curve $f(x)=\frac{2 x^{2}}{x^{2}-1}$.

1. Find the domain of $f$.
2. Find the $x-$ and $y$-intercepts of $f$.
3. Determine whether the graph of $f$ is symmetric to $y$-axis or the origin.
4. Find the horizontal and vertical asymptotes of $f$.
5. Find the intervals on which $f$ is increasing or decresing.
6. Find the relative extrems of $f$.
7. Determine the concavity and points of inflection of $f$.
8. Combine the information gathered in steps $1-7$ to sketch the graph of $f$.

## Sketching - Algebraic Practice

1 Analyze and sketch the graph of $y=\frac{2 x^{2}-8}{x^{2}-16}$.

2 Analyze and sketch the graph of $f(x)=2 x^{5 / 3}-5 x^{4 / 3}$

