When are numerical approximation techniques useful?

Quick Check

AP 1998

Let f be a function with $f(1) = 4$ such that for all points on the graph of f, the slope is given by $\dfrac{3x^2 + 1}{2y}$.

1. Find the slope of the graph of f at the point where $x = 1$.
2. Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$. Why might this be good approximation?

Newton's Method

- Method for approximating the real zeroes of a function.
- Uses tangent lines to approximate the function near its $x-$intercepts.

Consider a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If the y values $f(a)$ and $f(b)$ differ in sign, then by the intermediate value theorem, $f(x)$ must pass the intermediate value of zero $(f(x) = 0)$ somewhere in (a, b).

Let's visualize with Geogebra Applet then derive the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ ourselves.

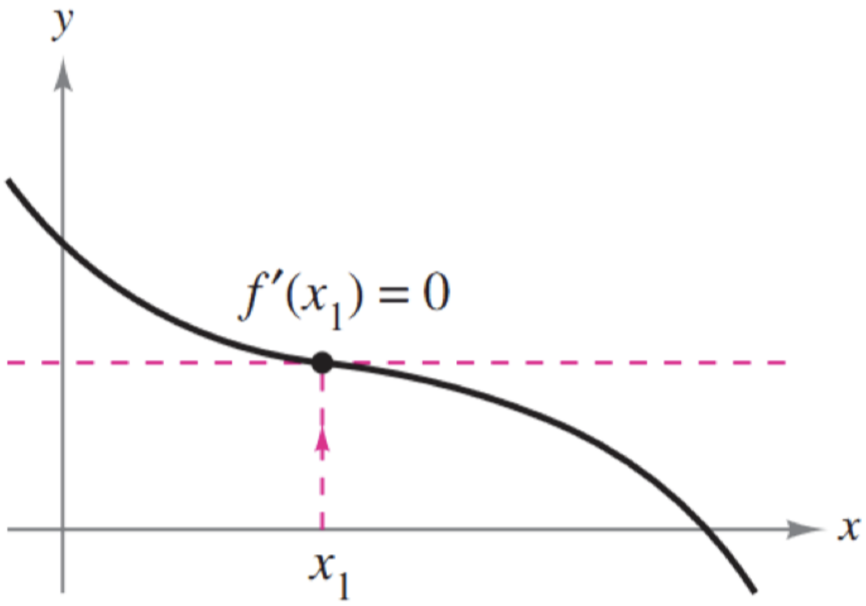
Using the Newton's Method

1️⃣ Use three iterations to approximate a zero of $f(x) = x^2 - 2$. Use $x_1 = 1$ as the initial guess.

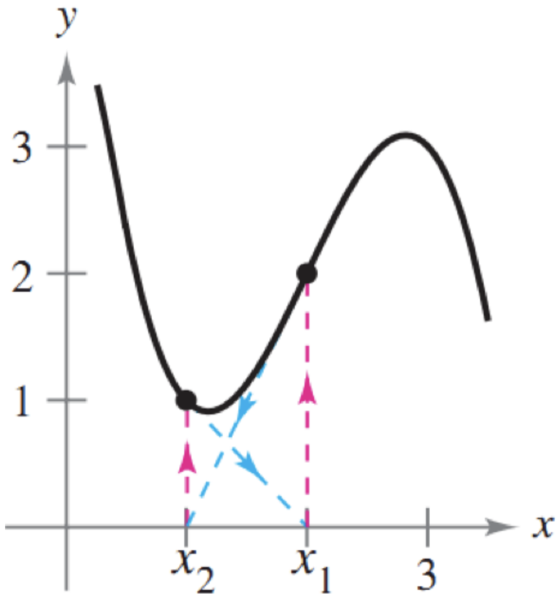
2️⃣ Use two iterations of Newton's Methods to approximate the zero of $f(x) = x^2 - 3$. Use $x_1 = 1.7$.

What to do when Newton's Methods Fails? 😱

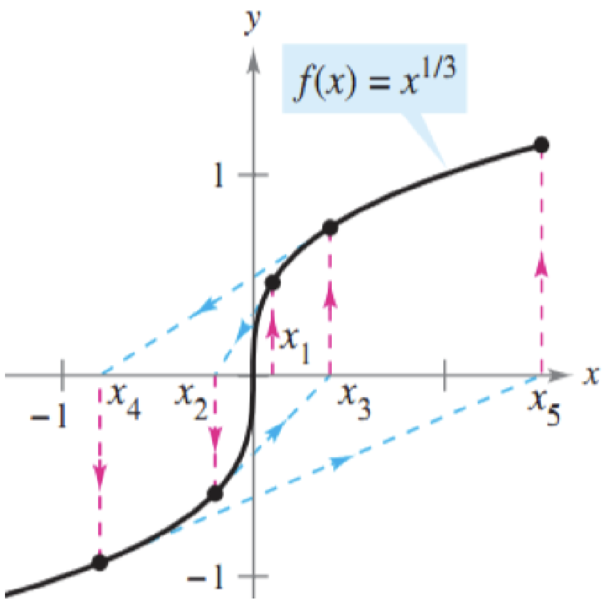
⚠️ Notice the division by $f'(x_n)$ in the formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$



OR looping



OR going farther



More problem solving uses of Newton's Method

Approximate the $x-$value(s) of the indicated point of intersection of the graphs of $f(x) = 3 - x$ and $g(x) = \dfrac{1}{x^2 + 1}$.