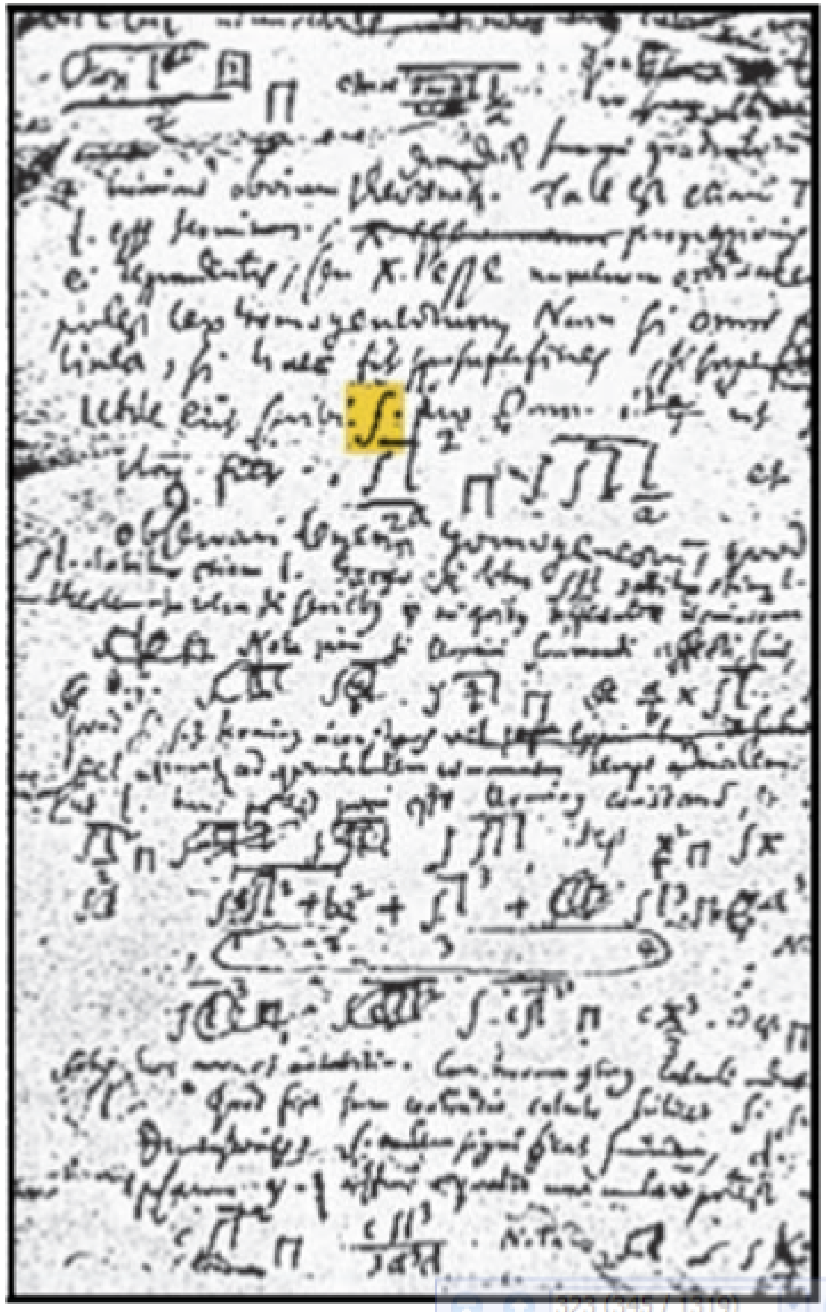
What is an antiderivative?

Quick Check

Find the derivative of each of the following functions.

1. $f(x) = x^3 - 3x^2 + 5$
2. $g(x) = x^3 - 3x^2 + \pi$
3. $h(x) = x^3 - 3x^2 - 1$
4. $p(x) = x^3 - 3x^2 - \sqrt{\pi}$

🤔 What conjecture can you make about the derivative of $f(x) = x^3 - 3x^2 + C$ where C is a constant?



Integral

Extract from the manuscript of Leibniz dated October 29, 1675 in which the integral sign first appeared.

Definition of the Antiderivative

A funciton F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Let $f(x) = 3x^2$ and $F(x) = x^3 + C$

$$F'(x) = f(x)$$

$$\frac{d}{dx}(x^3 + C) = 3x^2$$

The antiderivative of $3x^2$ is x^3 +C

Finding Antiderivatives

For each derivative, describe the original function F.

1. $F'(x) = 2x$
2. $F'(x) = x$
3. $F'(x) = x^2$
4. $F'(x) = \dfrac{1}{x^2}$
5. $F'(x) = \dfrac{1}{x^3}$
6. $F'(x) = \cos(x)$

🤔 What strategy did you use to find F?

The Indefinite Integral

Variable of integration

Constant of integration

$$y = \int f(x)\, dx = F(x) + C.$$

Integrand

antiderivative of f with respect to x

Using the definition of the antiderivative.

Derivative Formula	Equivalent Integration Formula
1 $\frac{d}{dx}[x^3] = 3x^2$	
2 $\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$	
3 $\frac{d}{dt}[\tan(t)] = \sec^2(t)$	
4 $\frac{d}{du}[u^{3/2}] = \frac{3}{2}u^{1/2}$	

Basic Integration Rules

$$\int F'(x)\,dx = F(x) + C$$

Differentiation is the "inverse" of integration.

if $\int f(x)\,dx = F(x) + C$, then

$$\frac{d}{dx}\left[\int f(x)\right] dx = f(x)$$

Integration is the "inverse" of differentiation.

Differentiation Formula	*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	

Integrating polynomial functions

1 Find the antiderivative of $f(x) = 3x$.

2 $\int dx$

3 $\int (x+2)\,dx$

4 $\int (3x^4 - 5x^2 + x)\,dx$

✨ Rewrite the integrand in a form that fits basic integration rules ✨

Original	Rewrite	Integrate
1 $\int \sqrt[3]{x}\,dx$		
2 $\int \frac{1}{x^2}\,dx$		
3 $\int \frac{1}{x\sqrt{x}}\,dx$		
4 $\int x(x^2+3)\,dx$		
5 $\int \frac{x^3+3}{x^2}\,dx$		
6 $\int \sqrt[3]{x}\,(x-4)\,dx$		

Find the indefinite integral and check your result by differentiation

1. $\int (x + x^2)\, dx$
2. $\int 4 \cos x \, dx$
3. $\int (3x^6 - 2x^2 + 7x + 1)\, dx$
4. $\int (x+1)(x^2 - 2)\, dx$
5. $\int \frac{t^2 - 2t^4}{t^4}\, dt$
6. $\int \frac{x+1}{\sqrt{x}}\, dx$
7. $\int \frac{\sin x}{\cos^2 x}\, dx$
8. $\int 2 \sin x + 3 \cos x \, dx$
9. $\int 1 - \csc t \cot t \, dt$
10. $\int (\tan^2 y + 1)\, dy$

Differential Equation

Find the general solution of $F'(x) = \dfrac{1}{x^2}, \quad x > 0.$

then find the particular solution that satisfies the initial condition $F(1) = 0.$

Visual Understanding

Identify which of the two graphs 1️⃣ and 2️⃣ is the graph of the funciton f and the graph of its antiderivative. Give a reason for your choice.