How do we use $\sum$ notation to write and evaluate sums?

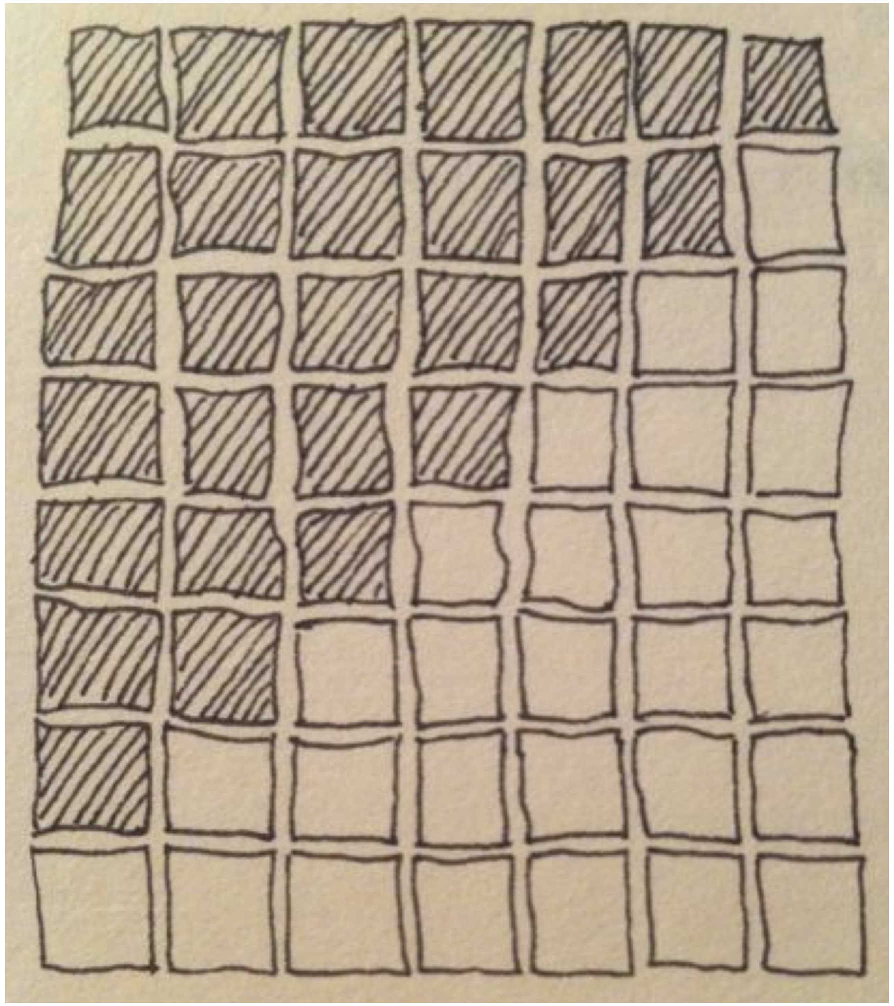
Quick Check

1. What is the sum of all integers from 1 to 100?
   $1+2+3+4+5+6+7+\ldots+98+99+100$

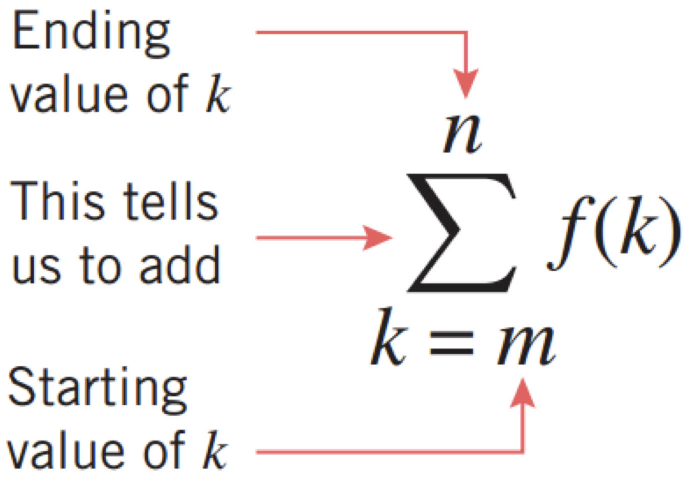
2. Generalize the result to find the sum of all integers from 1 to n?

🎥 Gauss - The Boy Genius, Prince of Maths

Counting the pebbles



Sigma Notation



$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \sum_{k=1}^{5} k^2$$

The summation of k^2,where k runs from 1 to 5.

Expand each sum (⚠️ Don't Simplify)

1 $\displaystyle\sum_{k=4}^{8} k^3$

2 $\displaystyle\sum_{k=1}^{5} 2k$

3 $\displaystyle\sum_{k=0}^{5} (2k+1)$

4 $\displaystyle\sum_{i=1}^{5} 2$

5 $\displaystyle\sum_{k=0}^{5} (-1)^k(2k+1)$

6 $\displaystyle\sum_{k=-3}^{1} k^3$

7 $\displaystyle\sum_{k=1}^{3} k\sin(\frac{k\pi}{5})$

8 $\displaystyle\sum_{j=0}^{2} x^3$

Rules of Summation

1. $\sum_{k=1}^{n} ca_k = c \cdot \sum_{k=1}^{n} a_k$ where c is a constant.

2. $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

3. $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

Use the rules of summation to expand each sum.

A $\sum_{k=1}^{10} 3k^2$

B $\sum_{k=2}^{8} (k + 3k^3)$

Summation Formulas

1. $$\sum_{i=1}^{n} c = cn$$

2. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

3. $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

4. $$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Evaluate $\displaystyle\sum_{i=1}^{n} \frac{i+1}{n^2}$ for $n = 10$ and $n = 10,000$.

Evaluate - Use the summation formulas to simplify each sum as a $f(n)$

1. $$\sum_{i=1}^{n} i(i-1)^2$$

2. $$\sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2$$

3. $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{16i}{n^2}$$

4. $$\lim_{n\to\infty} \sum_{i=1}^{n} (i+\frac{i}{n})(\frac{2}{n})$$