

How do we use \sum notation to write and evaluate sums?

Quick Check

1. What is the sum of all integers from 1 to 100?

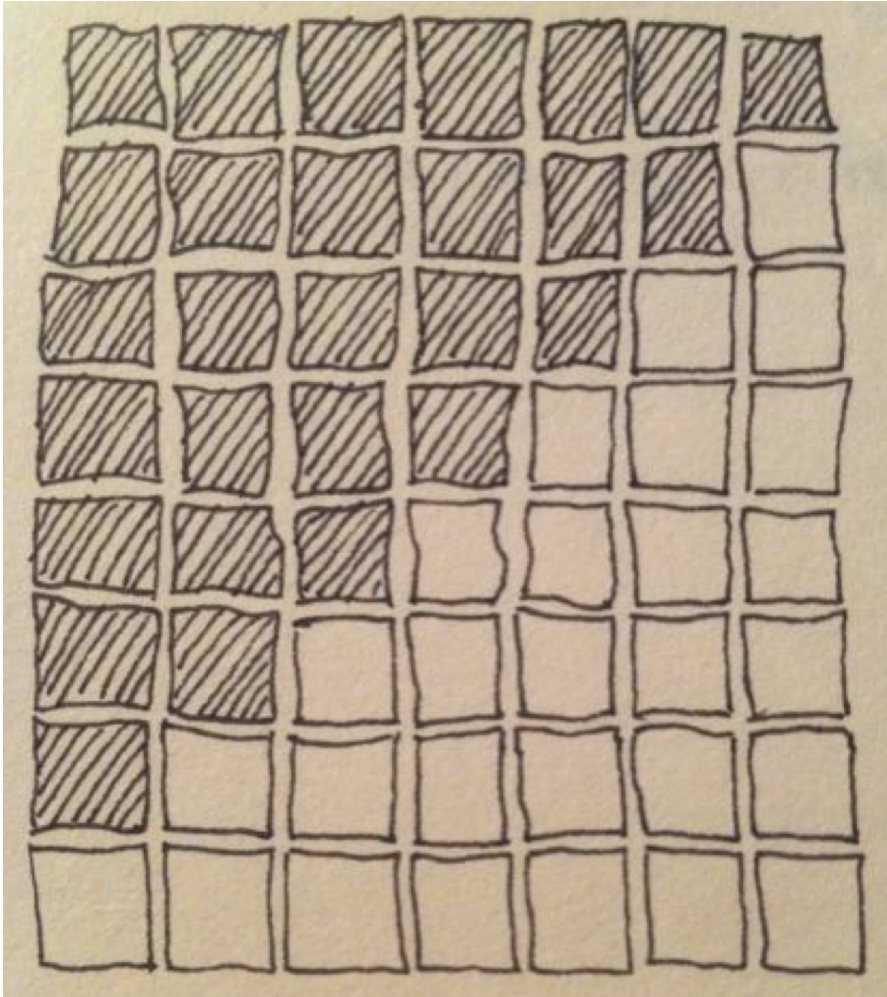
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots + 98 + 99 + 100$$

2. Generalize the result to find the sum of all integers from 1 to n ?



 Gauss - The Boy Genius,
Prince of Maths

Counting the pebbles



Sigma Notation

Ending value of k \rightarrow n

This tells us to add $\rightarrow \sum$

Starting value of k \rightarrow $k = m$

The diagram illustrates the components of the sigma notation $\sum_{k=m}^n f(k)$. Red arrows point from the text labels to the corresponding parts of the formula: from 'Ending value of k ' to n , from 'This tells us to add' to the summation symbol \sum , and from 'Starting value of k ' to $k = m$.

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \sum_{k=1}^5 k^2$$

The summation of k^2 , where k runs from 1 to 5.

Expand each sum (⚠ Don't Simplify)

$$1 \quad \sum_{k=4}^8 k^3$$

$$2 \quad \sum_{k=1}^5 2k$$

$$3 \quad \sum_{k=0}^5 (2k + 1)$$

$$4 \quad \sum_{i=1}^5 2$$

$$5 \quad \sum_{k=0}^5 (-1)^k (2k + 1)$$

$$6 \quad \sum_{k=-3}^1 k^3$$

$$7 \quad \sum_{k=1}^3 k \sin\left(\frac{k\pi}{5}\right)$$

$$8 \quad \sum_{j=0}^2 x^3$$

Rules of Summation

1. $\sum_{k=1}^n ca_k = c \cdot \sum_{k=1}^n a_k$ where c is a constant.

2. $\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$

3. $\sum_{k=1}^n (a_k - b_k) = \sum_{k=1}^n a_k - \sum_{k=1}^n b_k$

Use the rules of summation to expand each sum.

A $\sum_{k=1}^{10} 3k^2$

B $\sum_{k=2}^8 (k + 3k^3)$

Summation Formulas

$$\mathbf{1} \quad \sum_{i=1}^n c = cn$$

$$\mathbf{2} \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\mathbf{3} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\mathbf{4} \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Evaluate $\sum_{i=1}^n \frac{i+1}{n^2}$ for $n = 10$ and $n = 10,000$.

Evaluate - Use the summation formulas to simplify each sum as a $f(n)$

1
$$\sum_{i=1}^n i(i-1)^2$$

2
$$\sum_{i=1}^n \frac{1}{n^3} (i-1)^2$$

3
$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{16i}{n^2}$$

4
$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(i + \frac{i}{n}\right) \left(\frac{2}{n}\right)$$