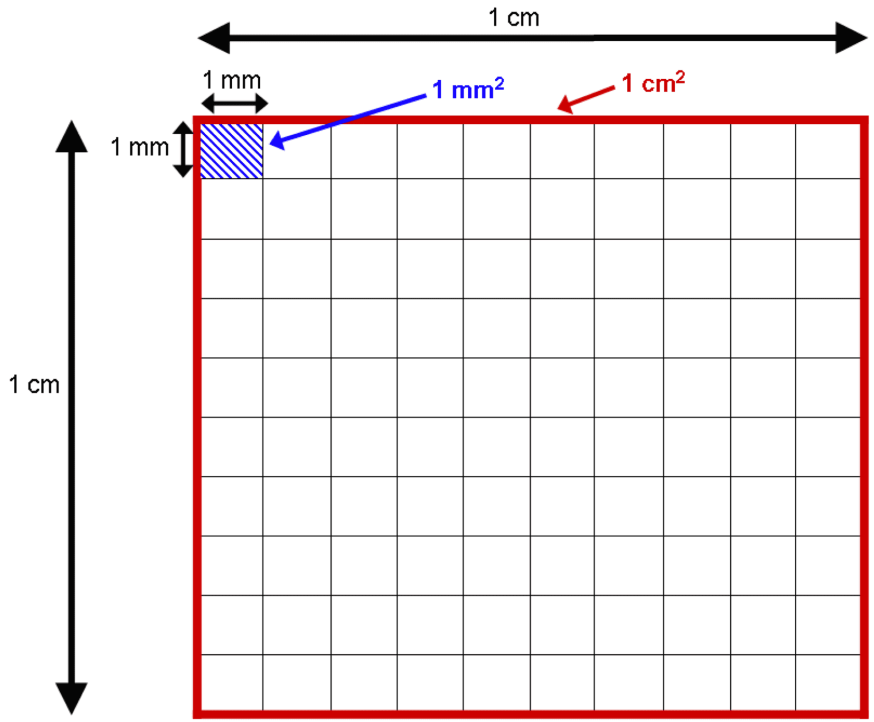
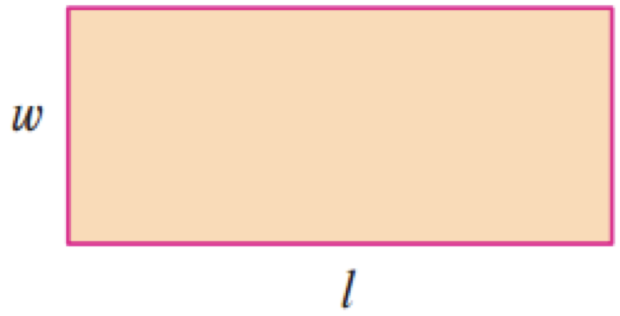
How do we find the area of a plane region using limits?

Quick Check

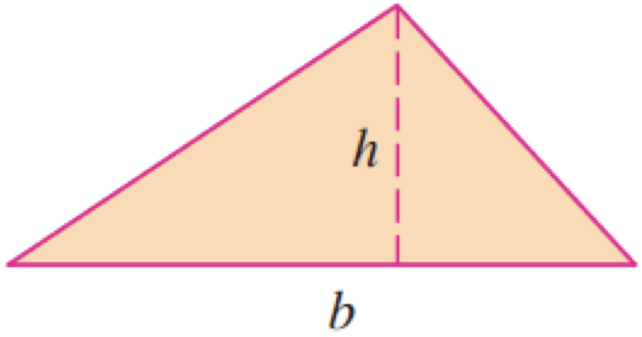
What is area?



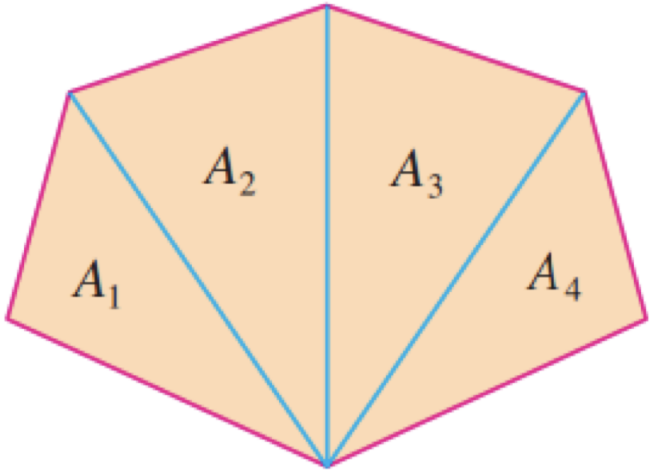
The Area Problem



$A = lw$



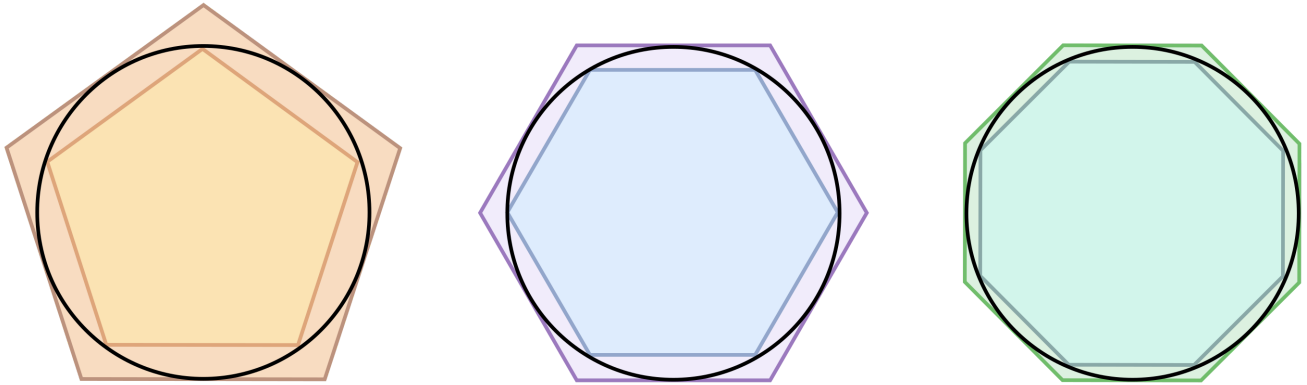
$A = \frac{1}{2}bh$



$A = A_1 + A_2 + A_3 + A_4$

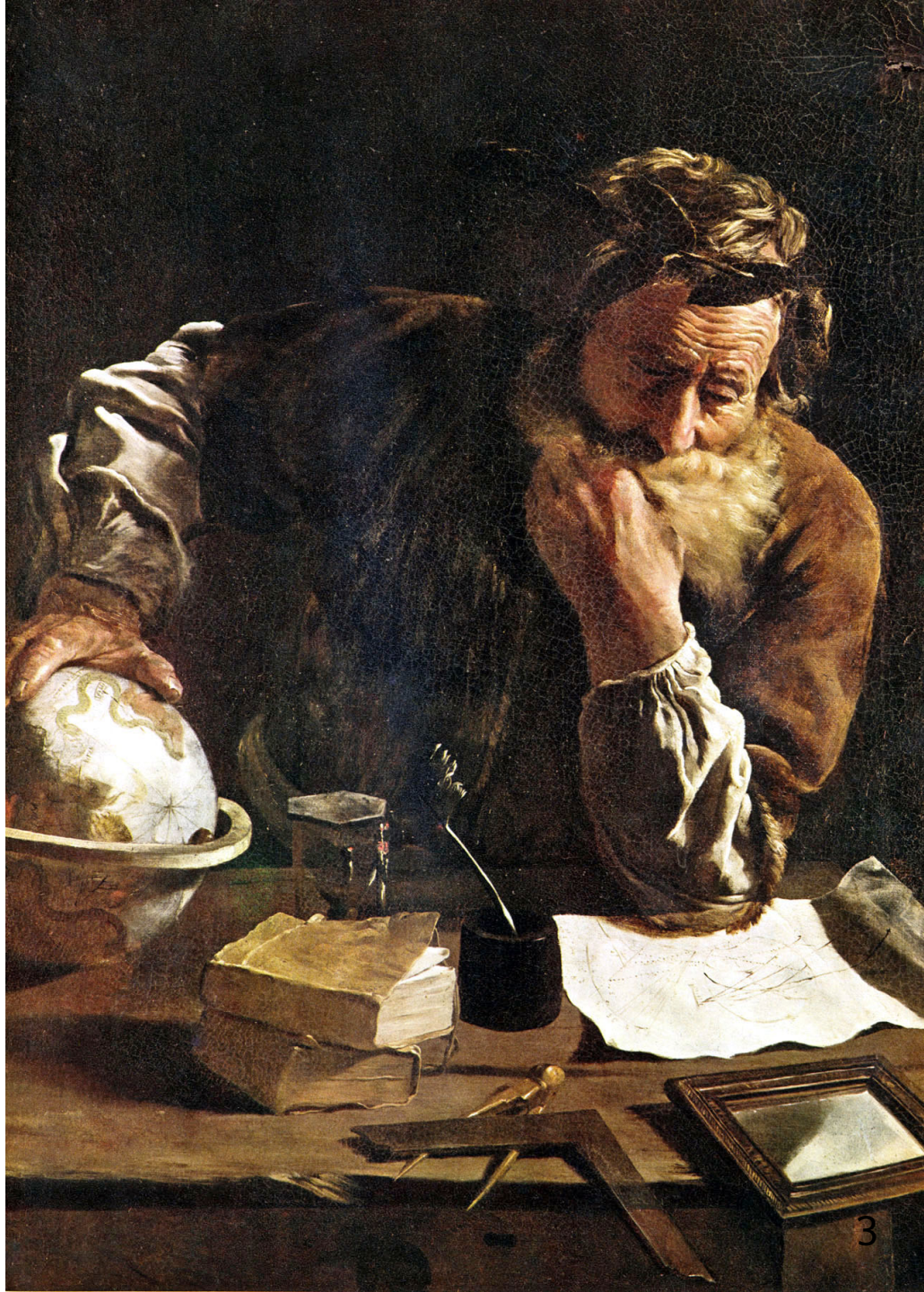
Finding the area of regions other than polygons is more difficult.

Method of Exhaustion



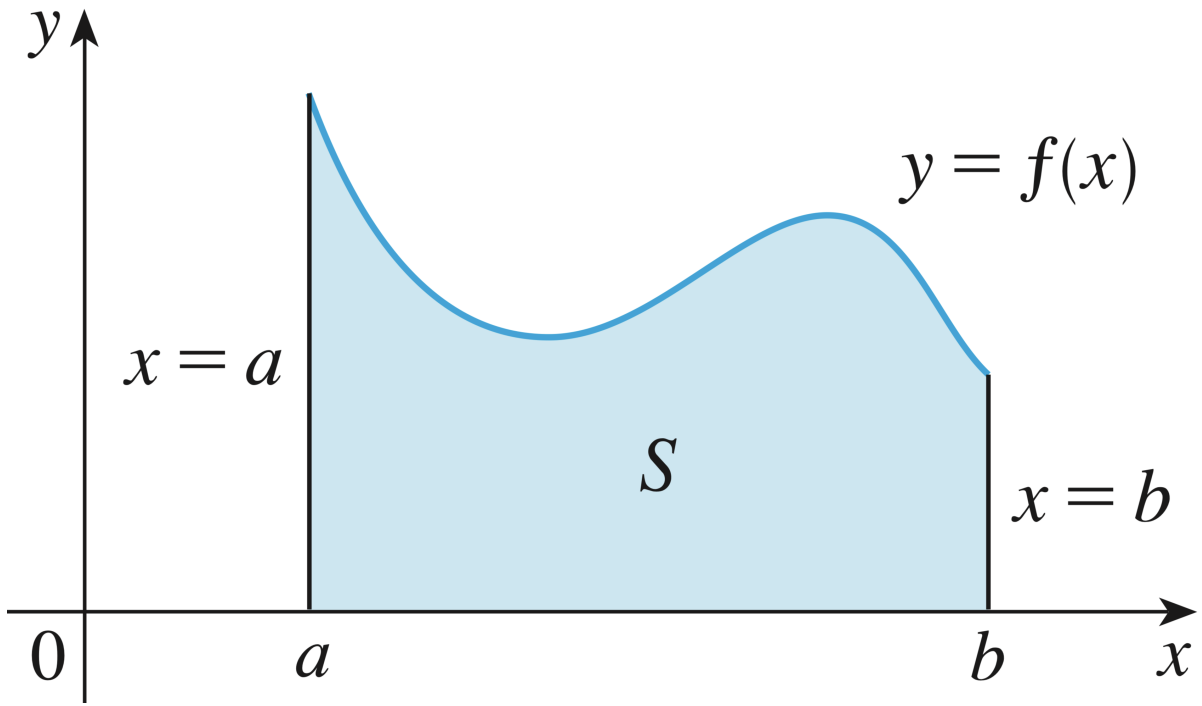
Archimedes used the method of exhaustion to compute the area inside a circle. (Wikipedia)

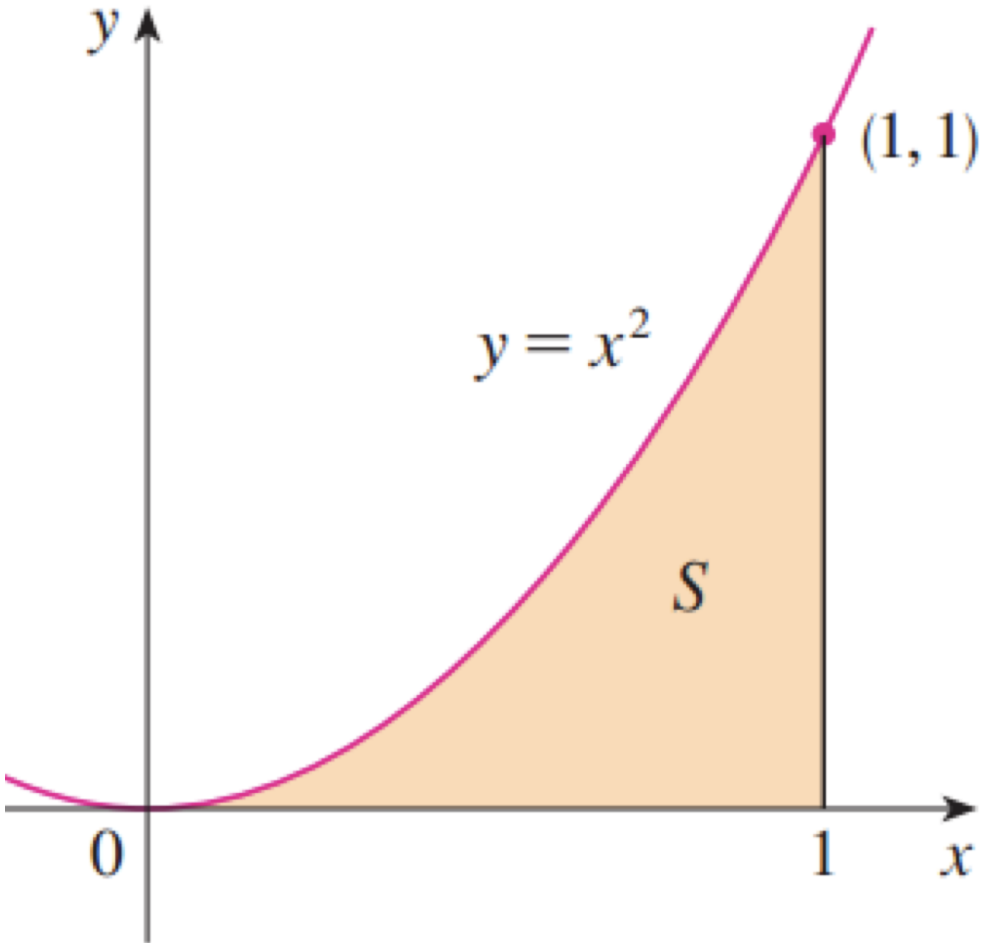
🎥 TED ED - Archimedes' Eureka!



Area under a curve

How can we compute the area of a region S that lies under the curve $y = f(x)$?





Approximating the area of a plane region

1. How many strips to cut the shape into?

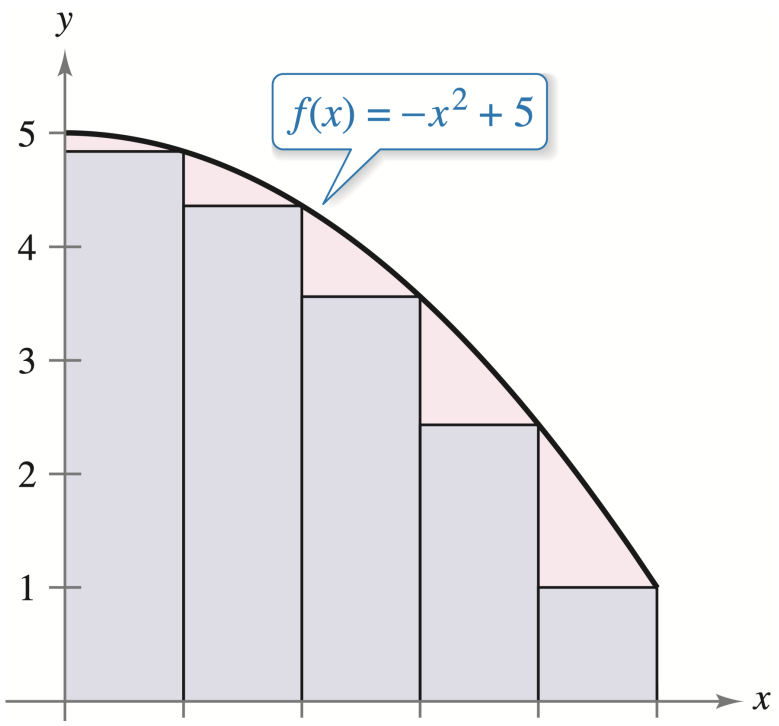
2. What will be the size of each cut?

🤔What decisions would keep the calculations easy?

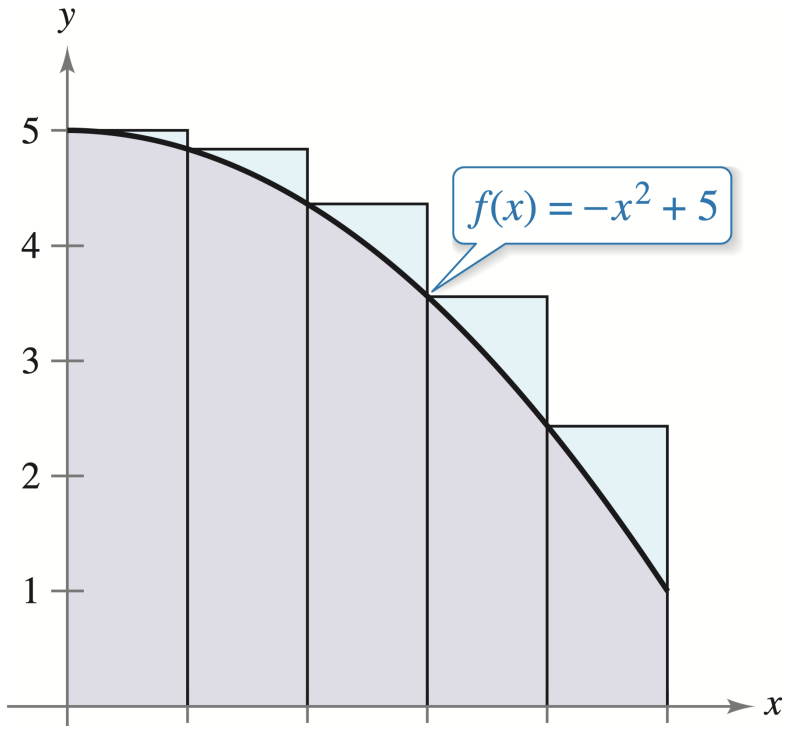
Practice

Approximate the area of the region lying between the graph of $f(x) = -x^2 + 5$ and the $x-$axis between $x = 0$ and $x = 2$. Use 5 rectangles shown compute each approximation.

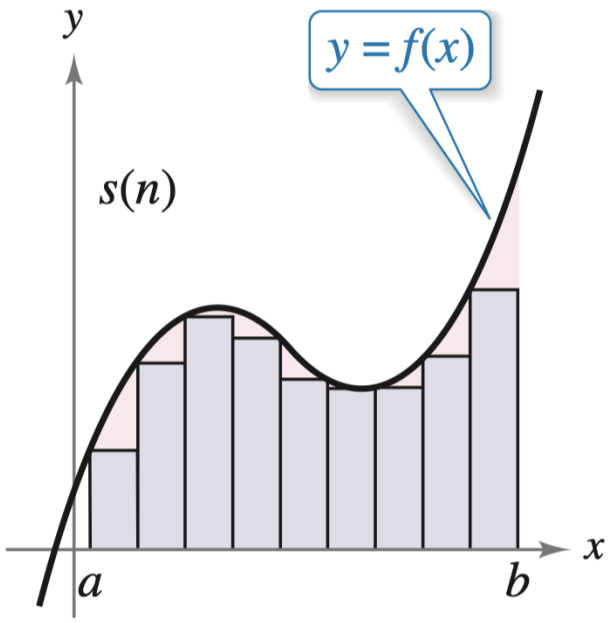
1

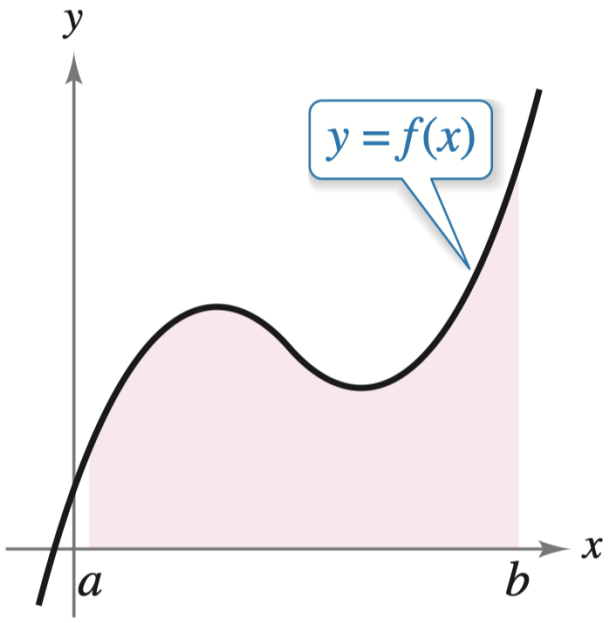


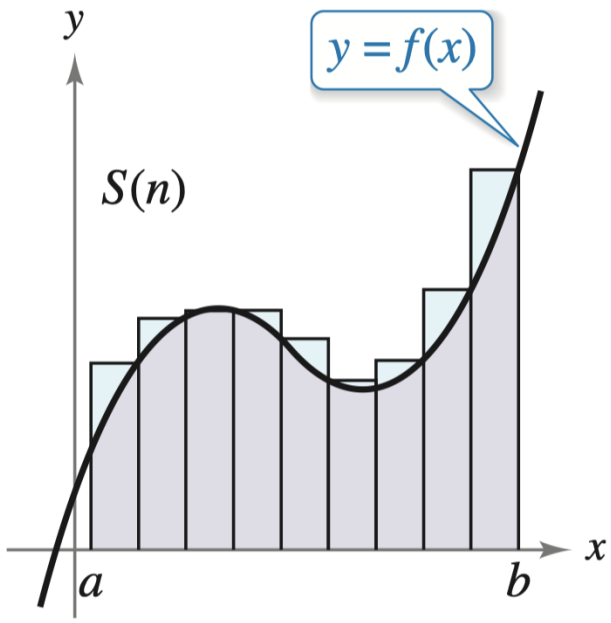
2



Upper Sum and the Lower Sum







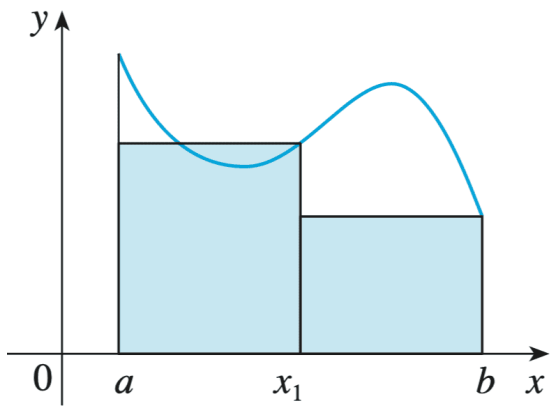
Rectangles formed using mimimum of the f on each subinterval create the lower sum.

Rectangles formed using maximum of the f on each subinterval create the upper sum.

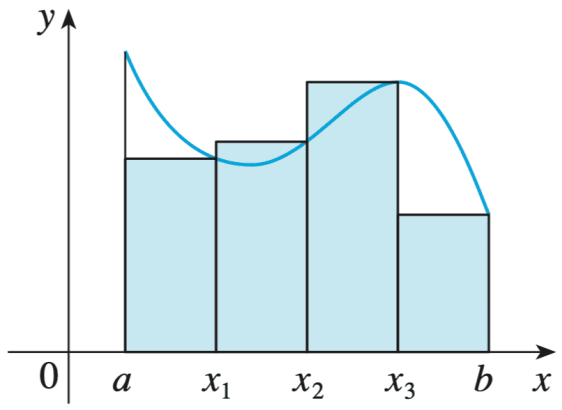
Definition of Area using Limits

The area of the region that lies under the graph of a continuous function f is the limit of the sum of the areas of approximating rectangles.

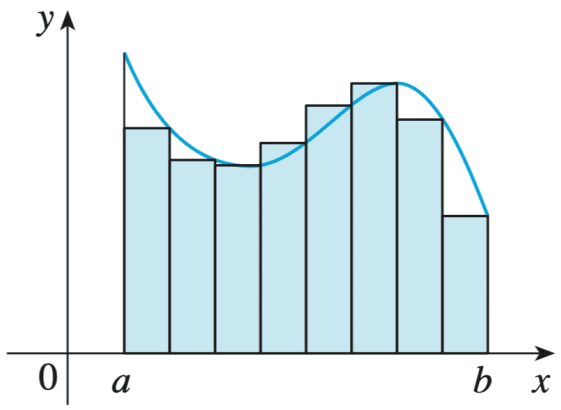
$$Area = \lim_{n\to\infty} \left[f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x \right]$$



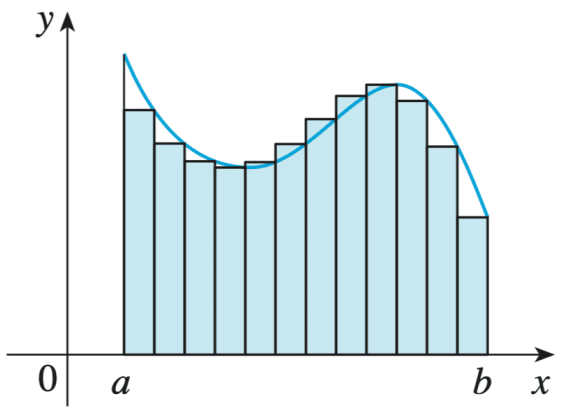
(a) $n = 2$



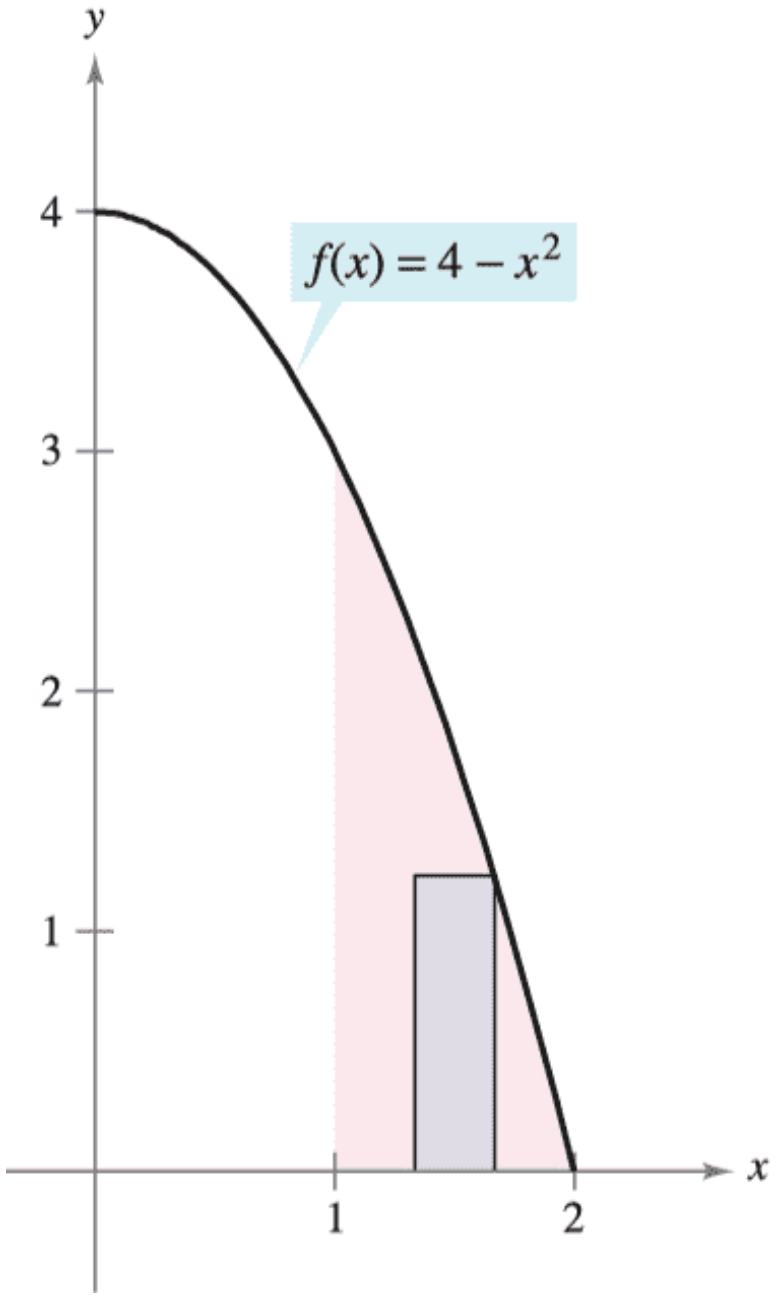
(b) $n = 4$



(c) $n = 8$



(d) $n = 12$



Finding Area using the Limit Definition

Find the area of the region bounded by the graph of $f(x) = 4 - x^2$, $x-$axis, and the vertical lines $x = 1$ and $x = 2$.