How does the Fundamental Theorem of Calculus connect the differential and integral calculus?

Quick Check

True or False? Explain.

1. $$\int_a^b f(x)g(x)\,dx = \left[\int_a^b f(x)\,dx\right]\left[\int_a^b g(x)\,dx\right]$$

2. The value of $\int_a^b f(x)\,dx$ must be positive.

Antidifferentiation and Definite Integration

$$\int f(x)\,dx$$

Antiderivative

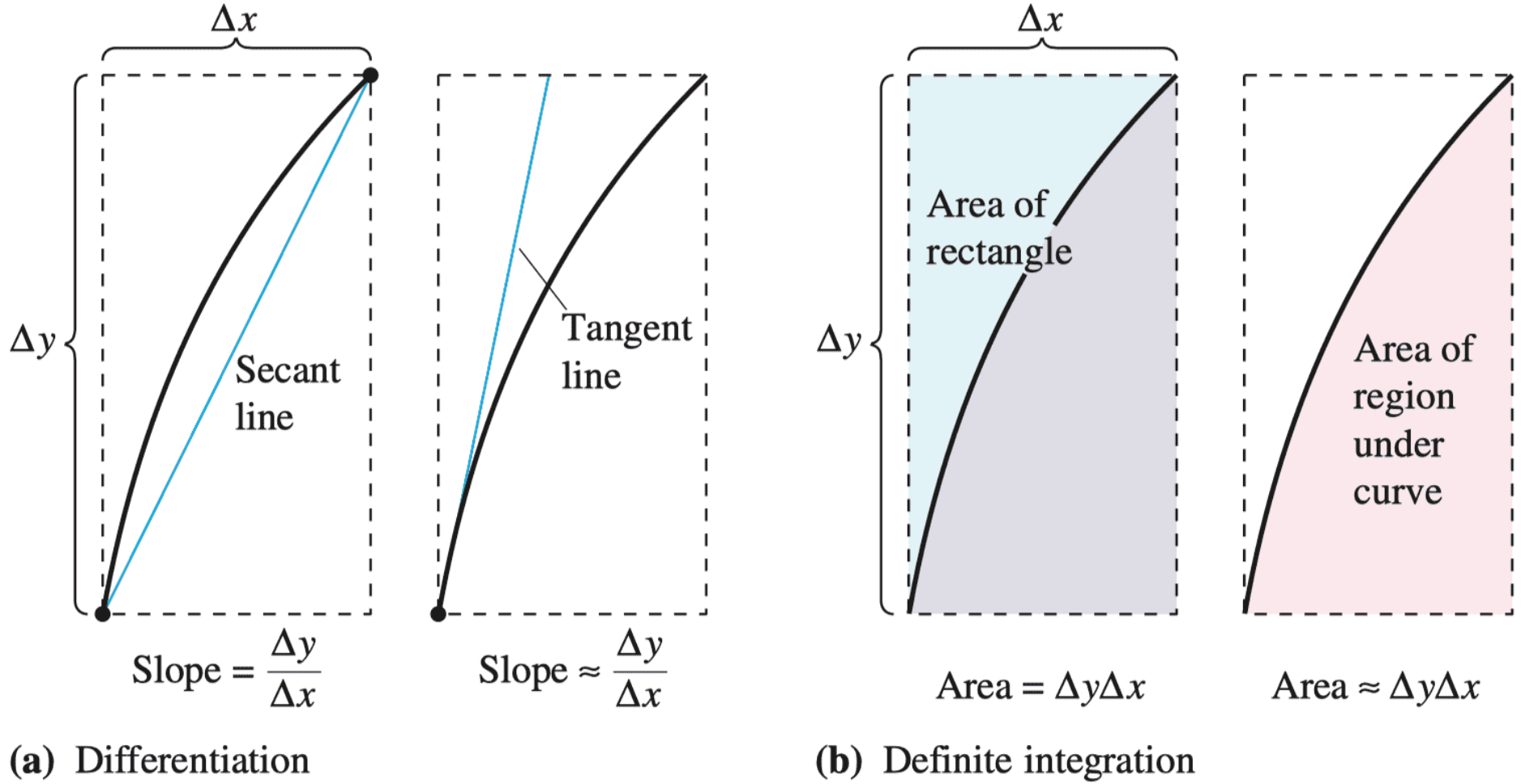
A family of functions

$$\int_a^b f(x)\,dx$$

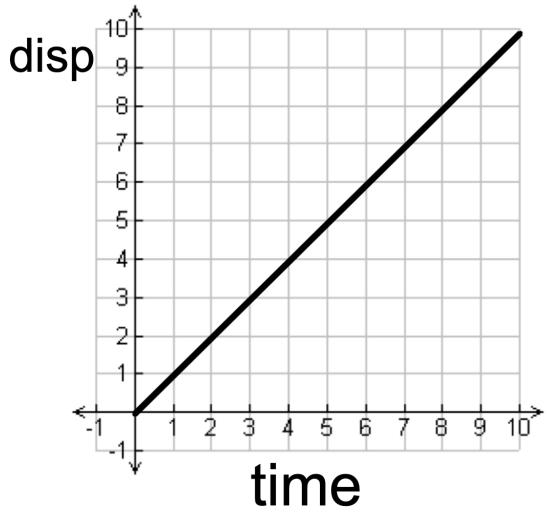
Definite Integral

A number

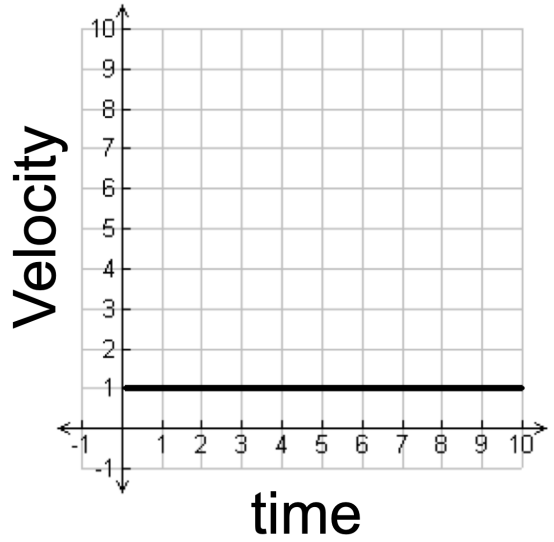
Inverse Relationship



Inverse Relationship



What is the total distance travelled by the particle from time t = 0 to t=10 ?



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The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Example:

$$\int_1^2 x\,dx$$

Evaluating a Definite Integral

1 $\int_1^2 (x^2 - 3)\,dx$

2 $\int_1^4 3\sqrt{x}\,dx$

3 $\int_0^{\pi/4} \sec^2(x)\,dx$

4 Find the area of the region bounded by $f(x) = x - x^2$, $x = 0$,$x = 1$, and the $x-$axis. Start by sketching the region.

5 $\int_0^2 |2x - 1|\,dx$

Evaluate each Integral

1. $\int_{-1}^{2} (x^3 - 2x)\, dx$
2. $\int_{1}^{4} (5 - 2t + 3t^2)\, dt$
3. $\int_{1}^{8} (\sqrt[3]{x})\, dx$
4. $\int_{1}^{2} \frac{3}{t^4}\, dt$
5. $\int_{1}^{2} \frac{s^4 + 1}{s^2}\, ds$
6. $\int_{0}^{1} (3 + x\sqrt{x})\, dx$
7. $\int_{0}^{\pi/4} (\sec(\theta)\tan(\theta))\, d\theta$
8. $\int_{-1}^{2} (x^3 - 2x)\, dx$
9. $\int_{0}^{3} |x^2 - 4|\, dx$

Explain the error

What is wrong with the equation? Explain.

$$\int_{-2}^{1} x^{-4}\,dx = \frac{x^{-3}}{-3}\Bigg|_{-2}^{1} = -\frac{3}{8}$$