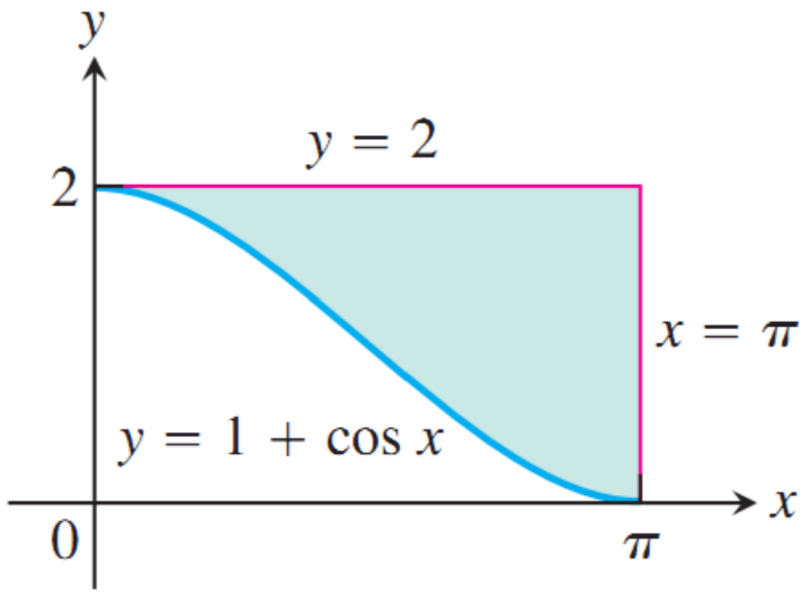
# What does the Mean Value Theorem for Integrals say?

## Quick Check

Find the area of the shaded region.



# Recall the Mean Value Theorem for derivatives

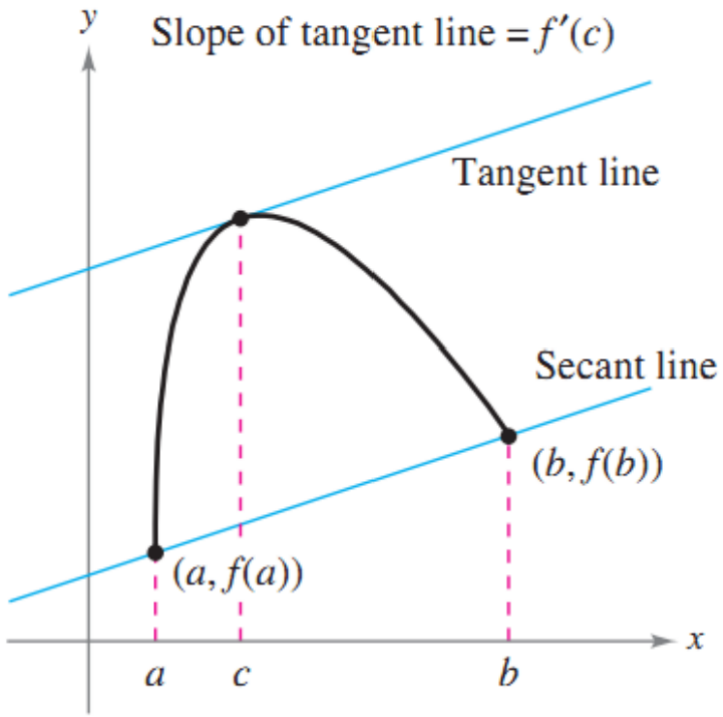
Let $f$ be a function that satisfies the following hypothesis:

- $f$ is continuous on the closed interval $[a, b]$
- $f$ is differentiable on the closed interval $(a, b)$

Then there is a number $c$ in the open interval $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\text{Instantaneous Rate of Change} = \text{Average Rate of Change}$$
$$\text{Slope of tangent at some point} = \text{Slope of secant line through endpoints}$$



## Recall

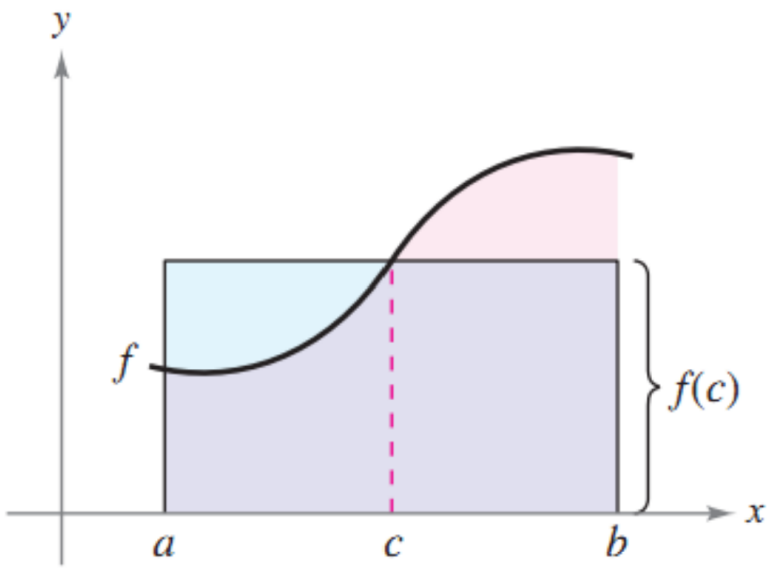
Can the Mean Value Theorem be applied? If so, find all $c$'s guaranteed by the theorem.

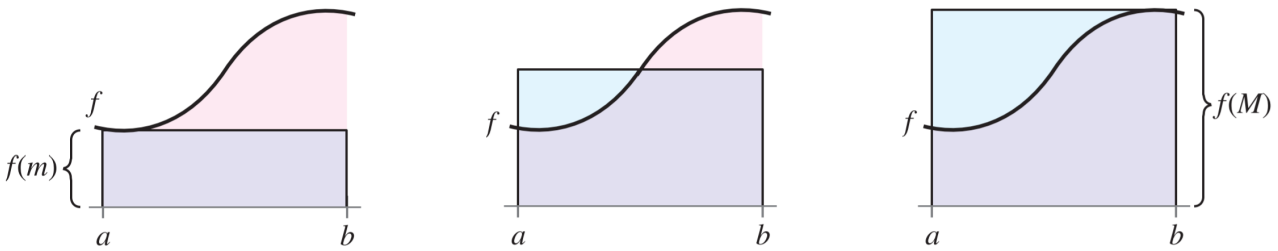
$$f(x) = x^2 \quad \text{on} \quad [-2, 1]$$

# Mean Value Theorem for Integrals

If $f$ is continuous on the closed interval $[a, b]$, then there exists a number $c$ in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$




Inscribed rectangle (less than actual area) | Mean value rectangle (equal to actual area) | Circumscribed rectangle (greater than actual area)

## Apply MVT

Find the value of $c$ guaranteed by the MVT for integrals for $f(x) = 4 - 2x$ on the interval $[0, 2]$.

## Average Value of a function

If $f$ is integrable on the closed interval $[a, b]$, then the average value of $f$ on the interval is

$$f(c) = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx$$

**Example:**

Find the average value of $f(x) = 4 - x$ on $[0, 3]$ and where $f$ actually takes on this value at some point in the given domain.

# Applying MVT for Integrals

**Find the average value of each function on the given interval.**

1. $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$.

2. $f(x) = \sin(x)$ on the interval $[0, \pi]$

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**Find the value(s) of $c$ guaranteed by the Mean Value theorem for Integrals for the function over the given interval.**

3. $f(x) = x^2 + 2x$ on the interval $[0, 1]$

4. $f(x) = \cos(x)$ on the interval $\left[\frac{\pi}{3}, \frac{\pi}{3}\right]$