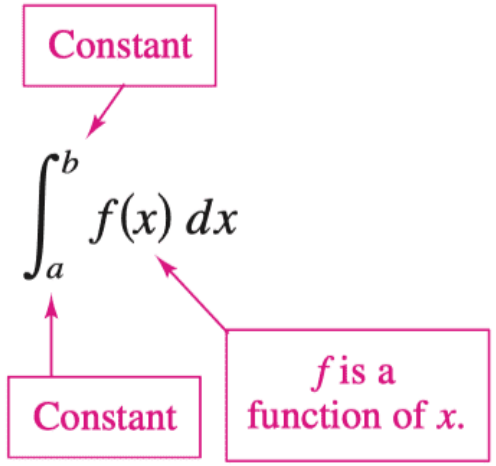
What is the Second Fundamental Theorem of Calculus?

Quick Check

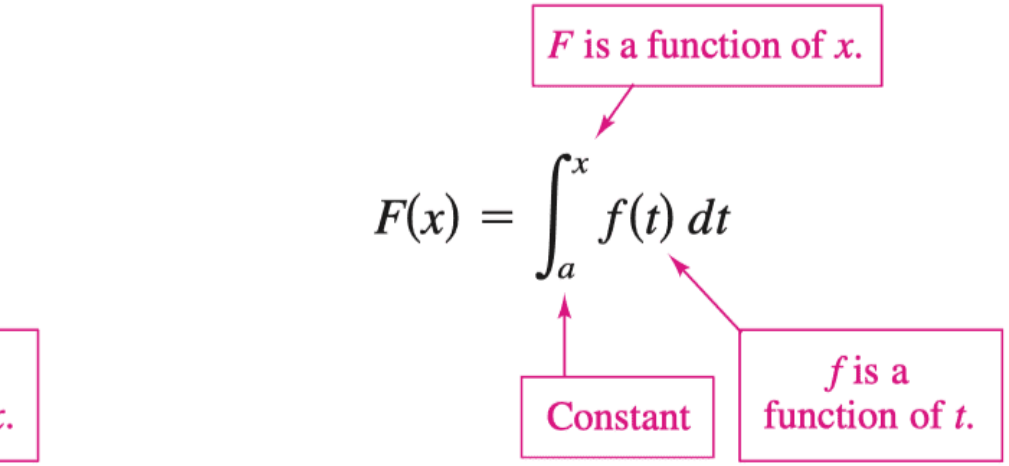
What is the difference between average rate or change and average value of a function?

Fundamental Theorem of Calculus (Part II)

The Definite Integral as a Number

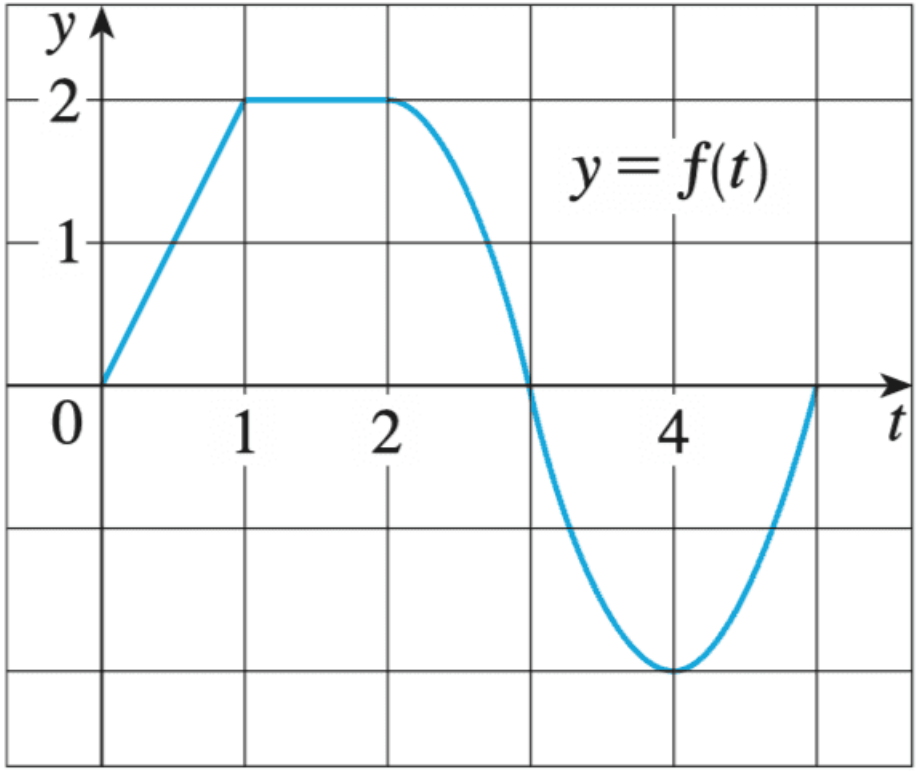


The Definite Integral as a Function of x

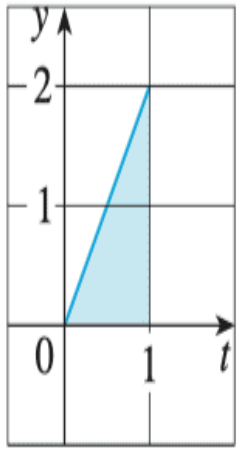


Accumulation Function

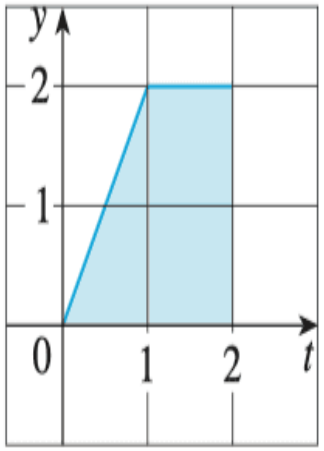
If f is the function whose graph is shown and $g(x) = \int_0^x f(t)dt$, find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, and $g(5)$. Then sketch a rough graph of g.



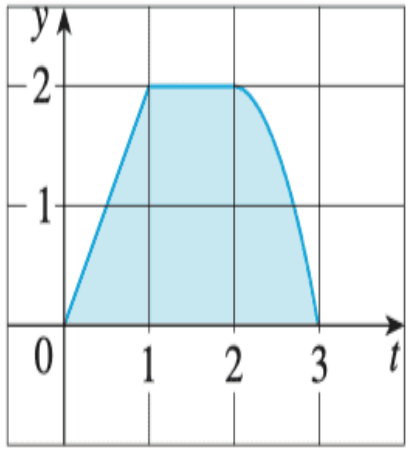
Textbook Visual



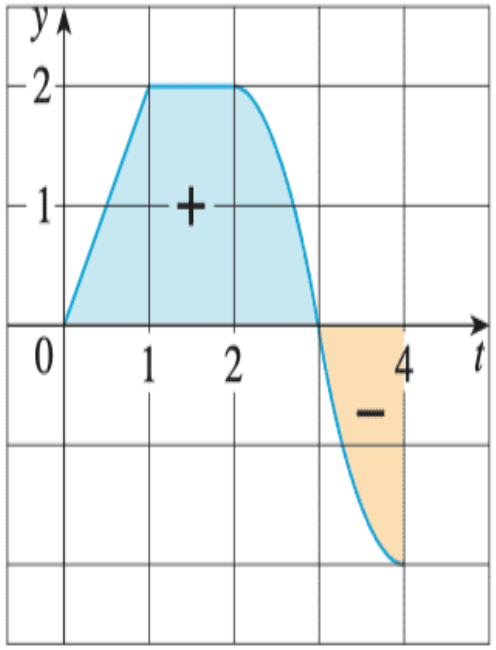
$g(1) = 1$



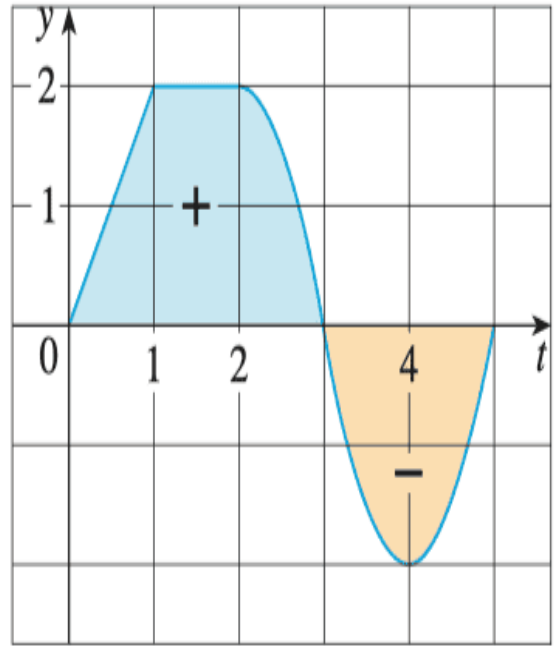
$g(2) = 3$



$g(3) \approx 4.3$



$g(4) \approx 3$



$g(5) \approx 1.7$

What would the graph of g look like?

Practice

Find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$.

1 $$F(x) = \int_0^x t - 5 \, dt$$

2 $$F(x) = \int_1^x \frac{10}{v^2} \, dv$$

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)\,dt = f\Big(p(x)\Big)\cdot p'(x)\right.$$

Example

1 $\dfrac{d}{dx}\left[\int_0^x \sqrt{t^2+1}\,dt\right]$

2 Find the derivative of $\displaystyle\int_{\pi/3}^{x^3} \cos(t)\,dt$

Practice

Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

1. $F(x) = \int_{-2}^{x} t^2 - 2t \, dt$

2. $F(x) = \int_{0}^{x} t\cos(t) \, dt$

3. $F(x) = \int_{0}^{x^3} t\sin(t^2) \, dt$

4. $F(x) = \int_{x}^{x+2} 4t + 1 \, dt$