How do we use $u - substitution$ technique to transform complicated integrals into simpler ones?

Quick Check

Apply the chain rule to find the derivative.

1 $F(x) = (4x - x^2)^{100}$

2 $\frac{d}{dx}\left[f\left(g(x)\right)\right]$

$u - substitution$

A

$$\int 3x^2(1+x^3)^{25}\,dx$$

What choice of u may allow easier integration?

$$\int u^{25}\,du$$

B

$$\int 2x\sin x^2\,dx$$

What choice of u may allow easier integration?

$$\int \sin u\,du$$

Chain Rule $\to$ pattern recognition

Our antidifferentiation formulas thus far don't tell us how to evaluate integrals such as

$$\int 2x\sqrt{1+x^2}\,du$$

$$\frac{d}{dx}\Big[F\big(g(x)\big)\Big] = F'\big(g(x)\big)\cdot g'(x)$$

Examples

1. $$\int 5\cos(5x)\,dx$$

2. $$\int x^2(x^3+2)^4\,dx$$

3. $$\int x^3\cos(x^4+2)\,dx$$

1. $$\int \frac{1}{(2x-4)^3}\,dx$$

2. $$\int \sin^2(3x)\cos(3x)\,dx$$

3. $$\int x\sqrt{2x-1}\,dx$$

4. $$\int (x+1)\sqrt{2x-1}\,dx$$

Practice

1 $$\int 3(3x-1)^4\,dx$$

2 $$\int (2x+1)(x^2+x)\,dx$$

3 $$\int 3x^2\sqrt{x^3-2}\,dx$$

1 $$\int \frac{-4x}{(1-2x^2)^2}\,dx$$

2 $$\int \cos^2(x)\sin(x)\,dx$$

Evaluating a definite integral using u-substitution technique

$$\int_0^1 x(x^2 + 1)^3 \, dx$$

Practice

1. $$\int_0^2 x\sqrt{x^2+4}\,dx$$

2. $$\int_0^{\pi/4} \cos^3(2x)\sin(2x)\,dx$$

3. $$\int_1^2 2x^2\sqrt{x^3+1}\,dx$$

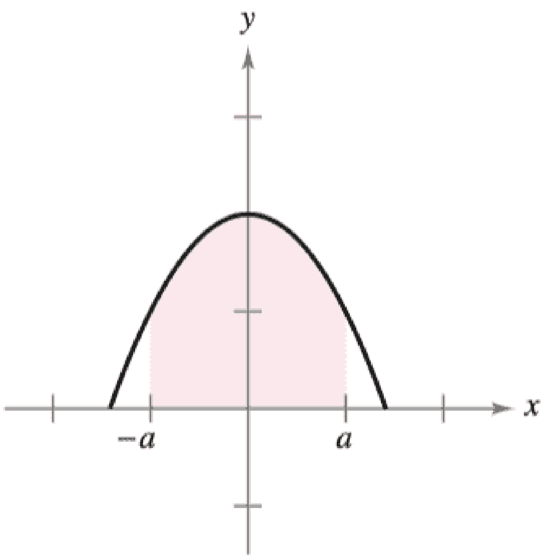
4. $$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$$

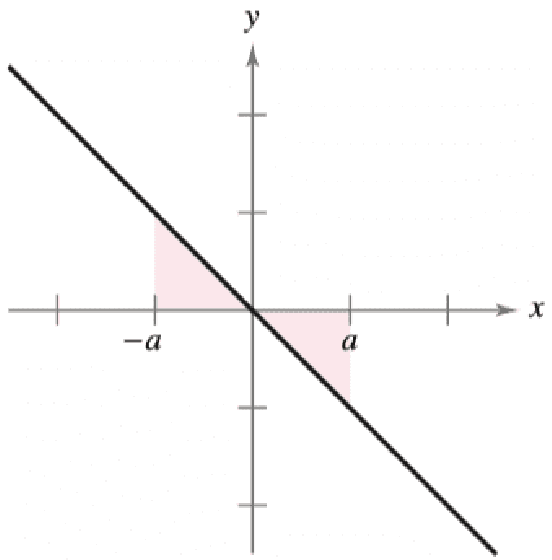
Integration of Even and Odd Functions

Suppose f is continuous on $[-a, a]$

A If f is even $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)\,dx = 2 \cdot \int_{0}^{a} f(x)\,dx$

B If f is odd $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$





Practice

Determine whether the following functions are even or odd

1. $f(x) = x^5 - x$
2. $f(x) = 2x - x^2$
3. $f(x) = \sin^2(x)\cos(x)$
4. $f(x) = x^2(x^2 + 1)$

Evaluate the following integrals of symmetric functions

A. $\int_{-2}^{2} x^2(x^2 + 1)\,dx$

B. $\int_{-\pi/2}^{\pi/2} \sin^2(x)\cos(x)\,dx$

Practice

1 $\int_{-2}^{2} (x^6 + 1)\, dx$

2 $\int_{-\pi/2}^{\pi/2} (\sin^3 x \cos x + \sin x \cos x)\, dx$