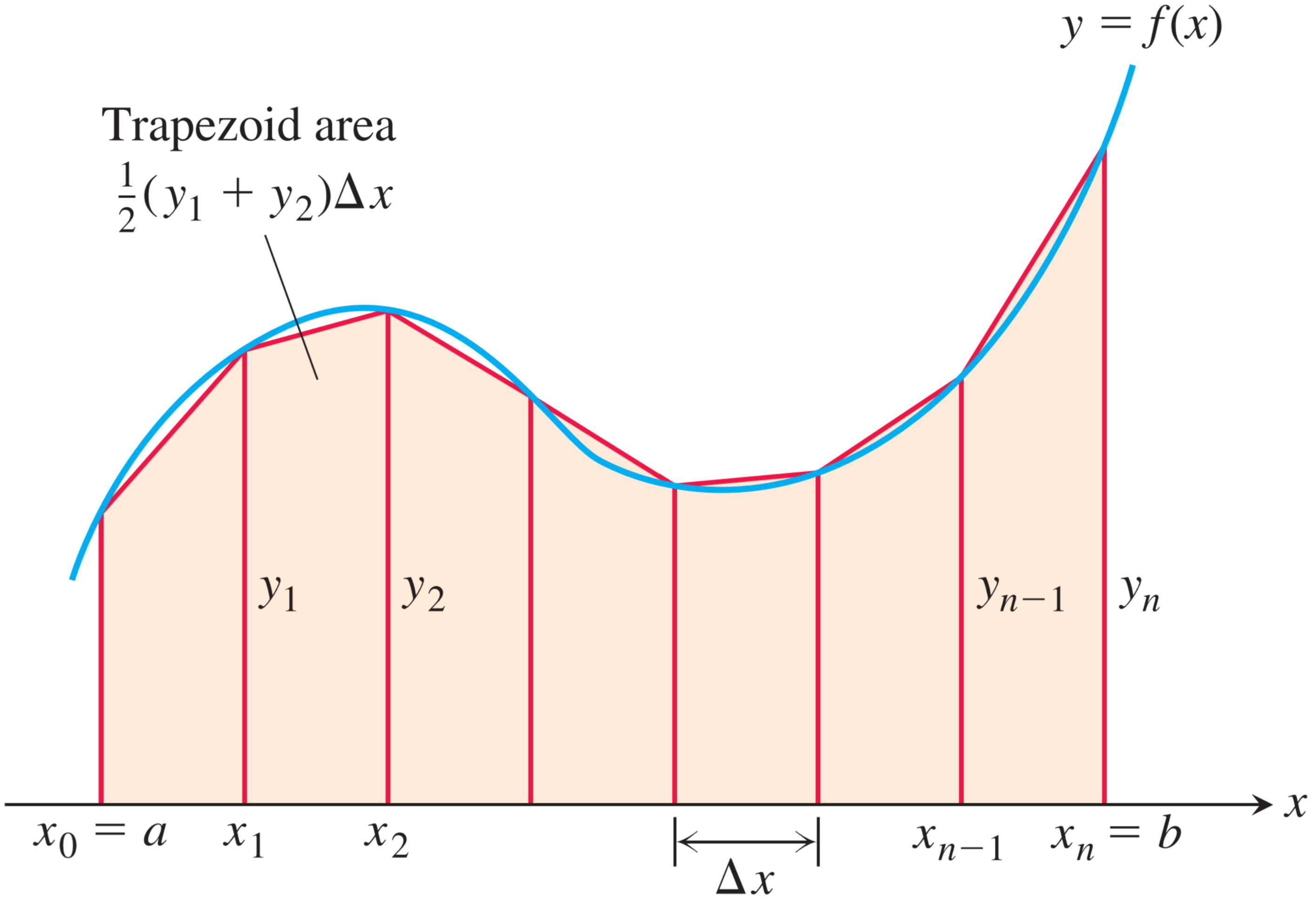
How can we use trapezoids to approximate the definite integral?

Quick Check

Illustrate the left, right, midpoint, upper, and lower Reimann Sums with sketches.
Draw atleast 4 rectangles in each sketch.

Trapezoidal Slices



Examples

1. Use 5 trapezoids to approximate the integral $\displaystyle\int_1^2 \frac{1}{x}\,dx$.

2. Use 4 trapezoids to approximate the integral $\displaystyle\int_1^2 x^2\,dx$. Use the fundamental theorem to compare the estimate to the exact value.

The Trapezoidal Rule

Let f be continuous on $[a, b]$.

$$\int_a^b f(x)\,dx \;\approx \frac{b-a}{2n}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)\Big]$$

As $n \to \infty$, the right hand side approaches $\int_a^b f(x)\,dx$

Practice

Use the trapezoidal rule to approximate the given definite integral for the given value of n. Round your answer to four decimal places. Compare your answer to the exact value of the definite integral.

1 $\int_0^2 x^3\,dx \qquad n = 4$

2 $\int_1^3 4 - x^2\,dx \qquad n = 4$

For question 2, find each of left, right, midpoint, upper, and lower Reimann Sums along with the Trapezoidal Sum and the exact value using the definite integral.