How can we approximate the definite integral using 2^{nd} degree polynomials (Simpson's Rule)?

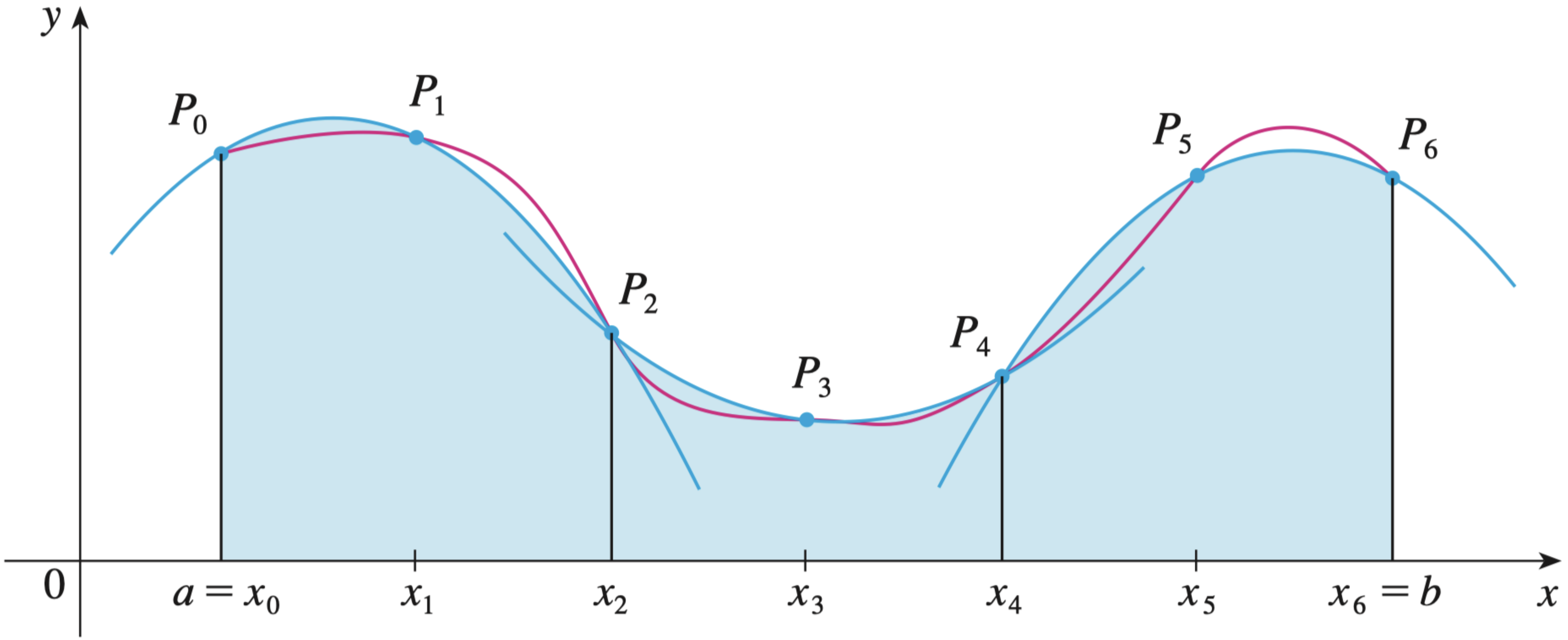
Quick Check

1 Determine whether the statement is TRUE or FALSE. Explain your answer.

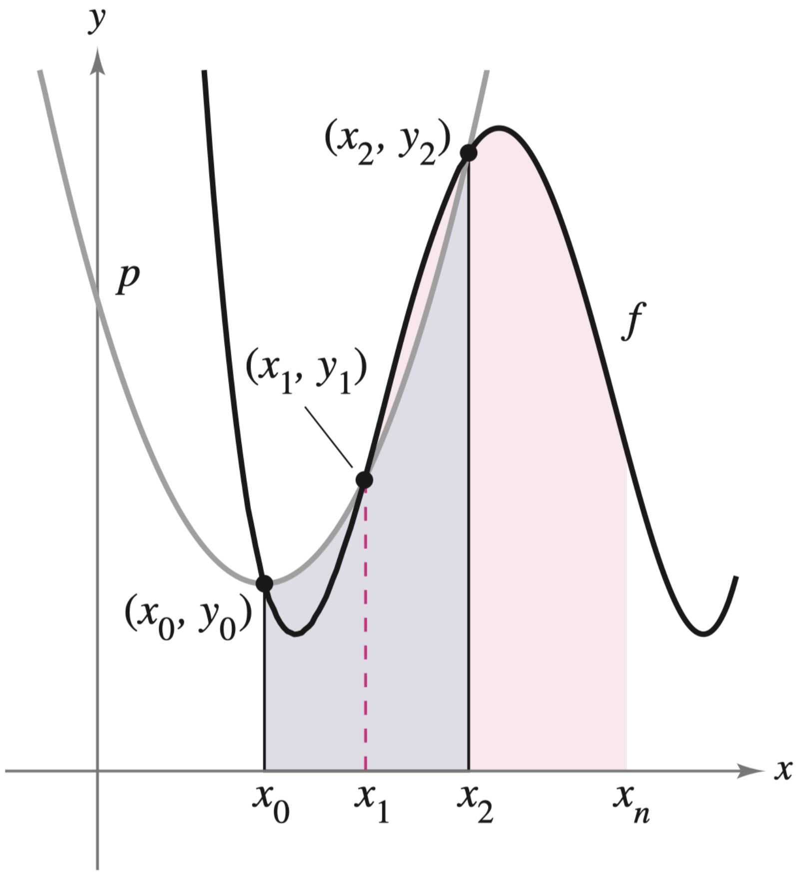
If $f(x)$ is concave down on the interval (a, b), then the trapezoidal approximation T_n underestimates $\int_a^b f(x)\,dx$.

2 Prove $\displaystyle\int_a^b x\,dx = \frac{b^2 - a^2}{2}$

Simpson's Rule



Simpson's Rule Intuition



$$\int_{x_0}^{x_2} p(x)\,dx \approx \int_{x_0}^{x_2} f(x)\,dx$$

Let $a = x_0$ and $b = x_2$

$$\int_a^b p(x)\,dx = \int_a^b Ax^2 + Bx + C\,dx$$

Simpson's Rule

Let f be continuous on $[a, b]$ and let n be an even integer. Simpson's Rule for approximating $\int_a^b f(x)\,dx$ is

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\big].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

Observe the function coefficients have a pattern $1 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \ldots 4 \quad 2 \quad 4 \quad 1$

Practice

Use the Trapezoidal Rule and the Simpson's Rule to approximate each definitte integral using n subintervals.

1 $\int_0^{\pi} \sin(x)\,dx$, $n = 4$

2 $\int_0^{8} \sqrt[3]{x}\,dx$, $n = 8$

2 $\int_2^{3} \frac{2}{x^2}\,dx$, $n = 4$

3 $\int_1^{4} (4 - x^2)\,dx$, $n = 6$