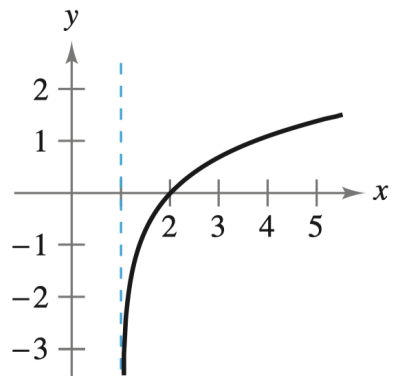
How do we find the derivatives of functions involving the natural logarithmic function?

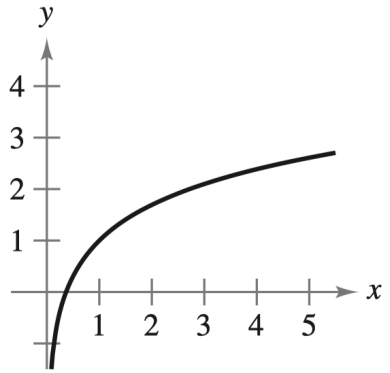
Quick Check

Match the function with its graph.

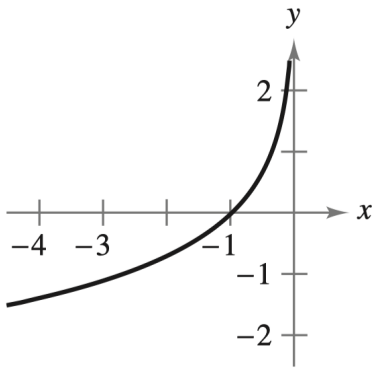
(a)



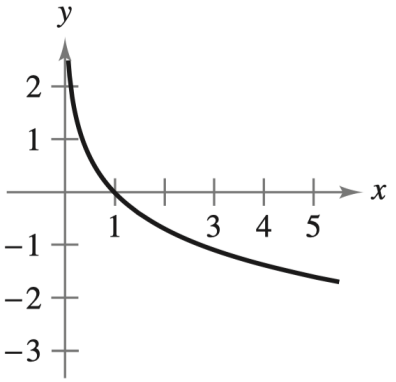
(b)



(c)



(d)



$f(x) = \ln x + 1$

$f(x) = -\ln x$

$f(x) = -\ln(-x)$

$f(x) = \ln(x - 1)$

Recall the 2nd Fundamental Theorem of Calculus

$$\frac{d}{dx}\left(\int_a^{p(x)} f(t)\,dt\right) = f\Big(p(x)\Big)\cdot p'(x)$$

Practice

1. Find the derivative of $\displaystyle\int_{\pi/2}^{x^3} \cos(t)\,dt$

2. $\displaystyle\frac{d}{dx}\left(\int_1^{x} t^3\,dt\right)$

Derivative of the Natural Logarithm Function

Let u be a differentiable function of x.

$$\frac{d}{dx}\Big[ln(u)\Big] = \frac{1}{u} \cdot \frac{du}{dx} \qquad u > 0$$

1 $\dfrac{d}{dx}\Big[\ln(2x)\Big]$

2 $\dfrac{d}{dx}\Big[\ln(x^2+1)\Big]$

3 $\dfrac{d}{dx}\Big[x\ln(x)\Big]$

4 $\dfrac{d}{dx}\Big[(\ln x)^3\Big]$

Use Log Properties to help with differentiation

Differentiate each function

1. $f(x) = \ln\left(\dfrac{x(x^2+1)^2}{\sqrt{2x^3-1}}\right)$

2. $f(x) = \ln\sqrt{x+1}$

Logarithmic Differentiation

Find the Derivative of each function

1. $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

2. $y = x\sqrt{x^2-1}$

Derivative involving Absolute Value

Let u be a differentiable function of x such that $u \neq 0$

$$\frac{d}{dx}\left[ln|u|\right] = \frac{1}{u} \cdot \frac{du}{dx}$$

Proof in two cases

1 $u > 0$

2 $u < 0$

Practice

Find the derivative of each function.

1. $f(x) = \ln|\cos x|$

2. $y = \ln|\sin x|$

Find the equation of the tangent line to the graph of f at the given point.

3. $f(x) = 3x^2 - \ln x \qquad (1, 3)$