How do we use the log rule of integration to integrate rational functions?

Quick Check

1. Find the equation of the tangent line to the graph of $y = \sin(2 \ln x)$ at $(1, 0)$.

2. Find $f'(x)$. View the graphs of f and f' on your calculator.
   $f(x) = \sin x + \ln x$

📣 Recall the definition of the anti-derivative.

$$\int x^2\,dx = \frac{x^3}{3} + C$$

because

$$\frac{d}{dx}\left(\frac{x^3}{3} + c\right) = x^2$$

Log Rule for Integration

The differentiation rule for natural logarithm is

$$\frac{d}{dx}\Big[ln(u)\Big] = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\downarrow$$

Using the definition of the antiderivative

Log Rule for Integration is...

Using the Log Rule for Integration

Examples

1. $\int \frac{10}{x}\,dx$
2. $\int \frac{1}{3x+2}\,dx$
3. $\int \frac{x^2}{3-x^3}\,dx$
4. $\int \frac{x}{\sqrt{9-x^2}}\,dx$

Practice

5. $\int \frac{1}{x-5}\,dx$
6. $\int \frac{3x^2+1}{x^3+x}\,dx$
7. $\int \frac{\sec^2 x}{\tan x}\,dx$
8. $\int \frac{1}{x\ln x}\,dx$

More Practice

9 Find the area of the region bounded by the graph of $y = \dfrac{x}{x^2+1}$, the $x-$axis, and the line $x = 3$. Start by using your calculator to sketch the region.

10 $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

Use Long Division before Integrating

Example

1. $\int \frac{x^2+x+1}{x^2+1}\,dx$
2. $\int \frac{x^3-6x-20}{x+5}\,dx$

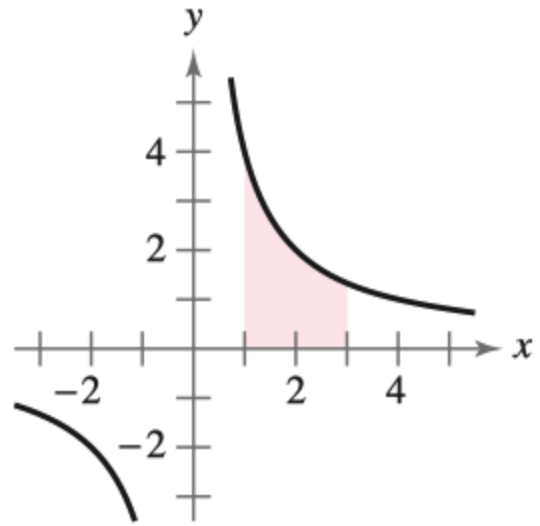
Practice

3. $\int \frac{x^4+x-4}{x^2+2}\,dx$
4. $\int \frac{x^3-3x^2+4x-9}{x^2+3}\,dx$

Practice

Find the area of the given region.

$$y = \frac{4}{x}$$



$$y = \frac{2}{x \ln x}$$