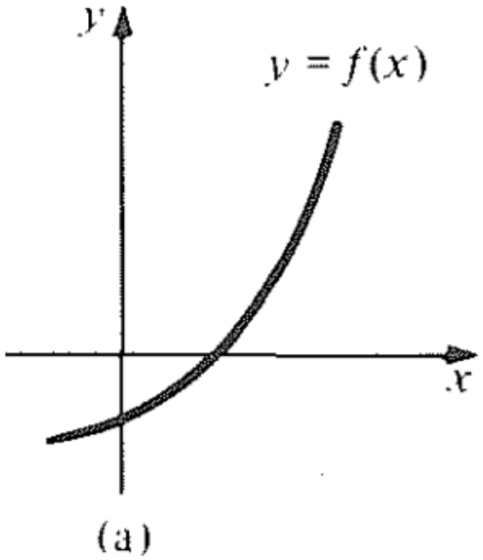
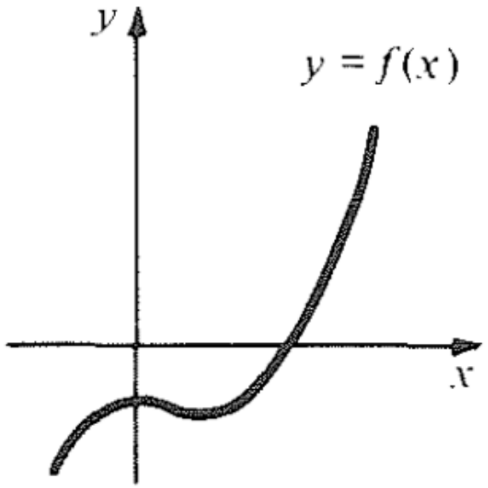
How do we find the derivative of an inverse function?

Quick Check

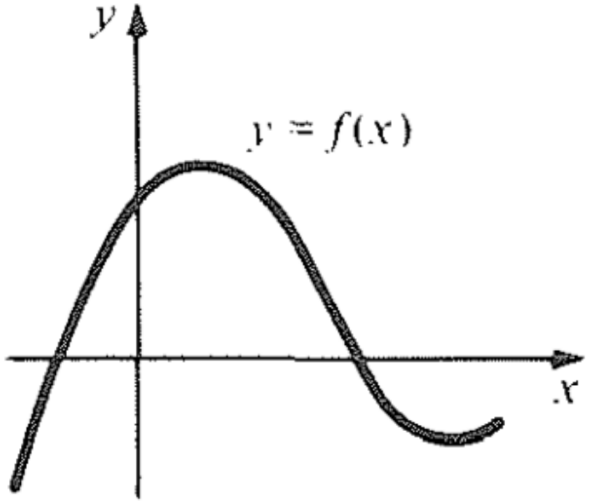
Determine whether each of the following functions have inverses. Explain.



(a)

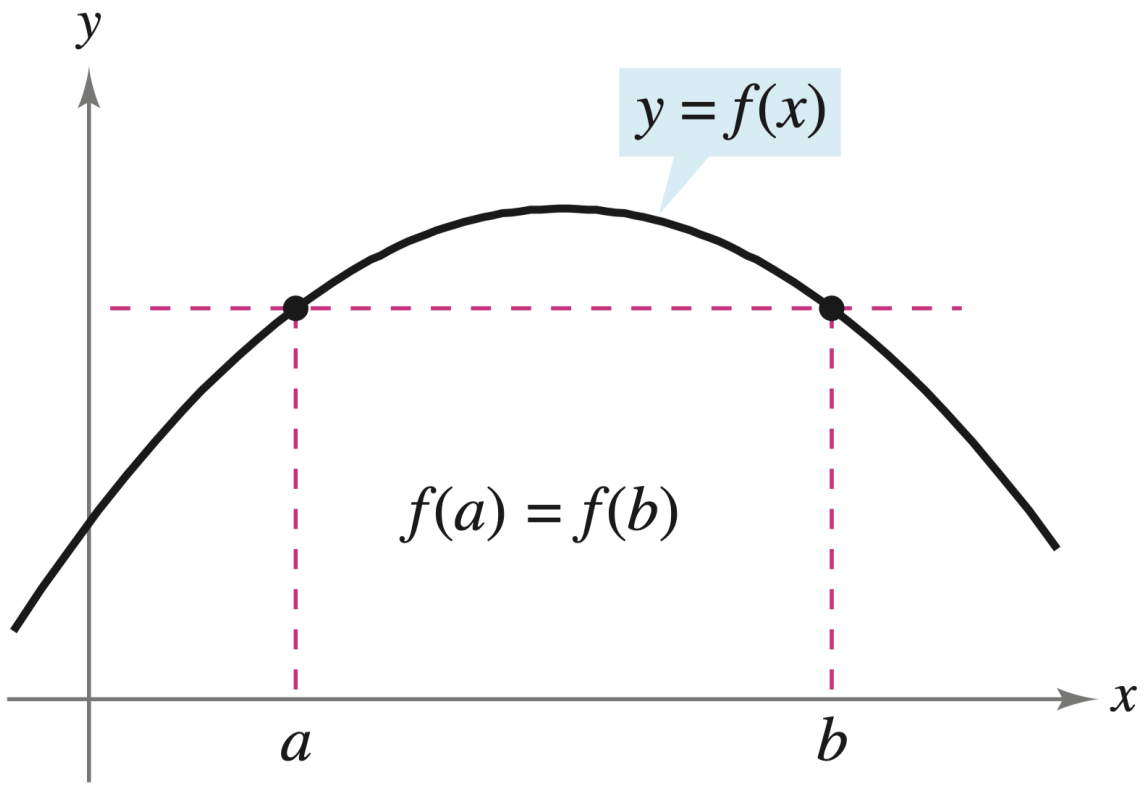


(b)

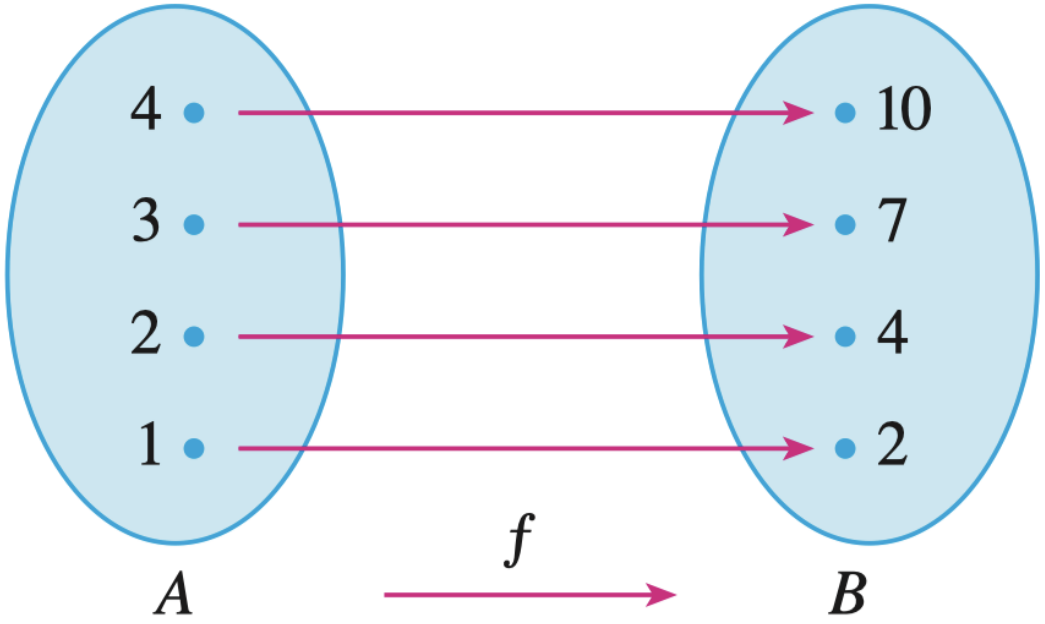


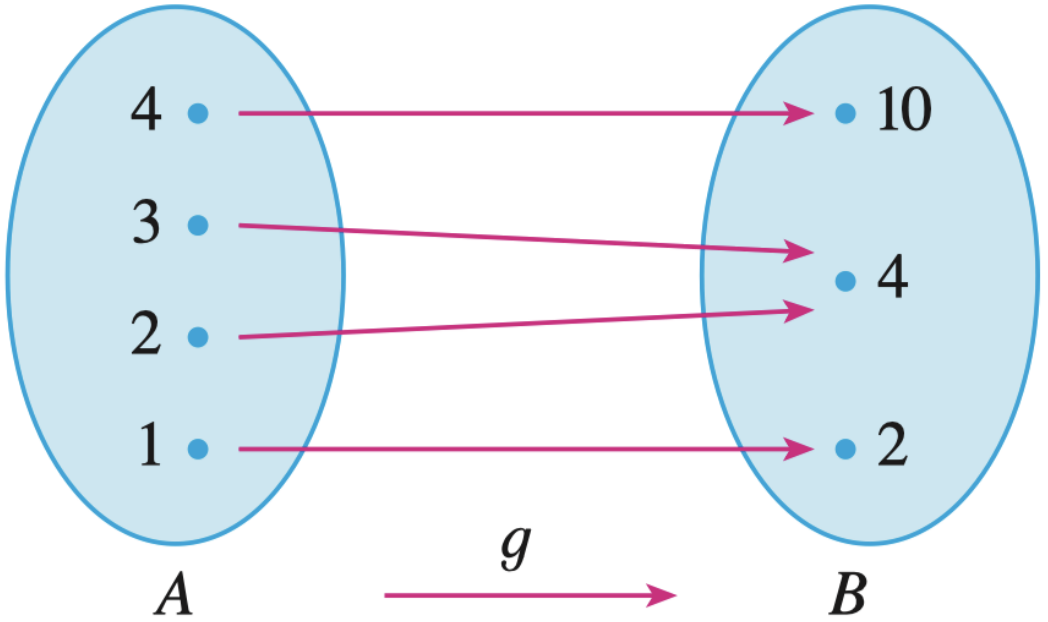
(c)

Horizontal Line Test



Inverse Function





The **inverse** of a function is a rule that acts on the output of the function an produces the corresponding input. So, the **inverse "undoes" or reverses what the function has done.** Not all functions have inverses; those that do are alled one-to-one.

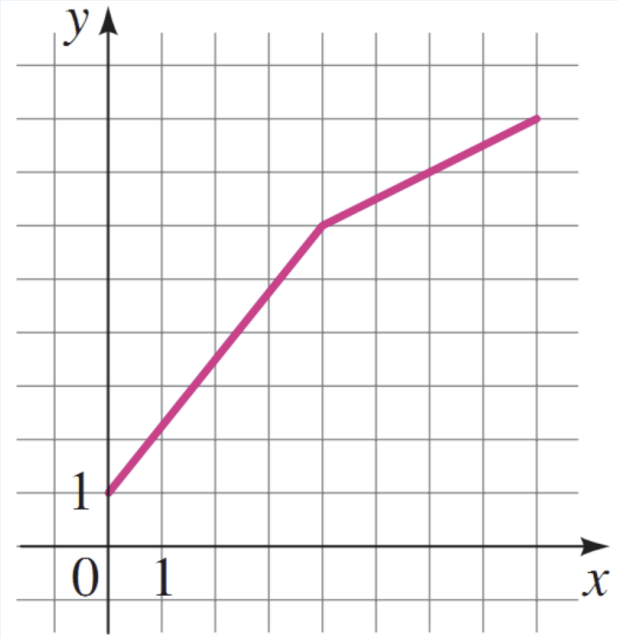
Notation for inverse functions

A Find f^{-1} for specified values.

If $f(1) = 5$, $f(3) = 7$, and $f(8) = -10$, find

1. $f^{-1}(5)$
2. $f^{-1}(7)$
3. $f^{-1}(-10)$

B Label the graph of f and its inverse.

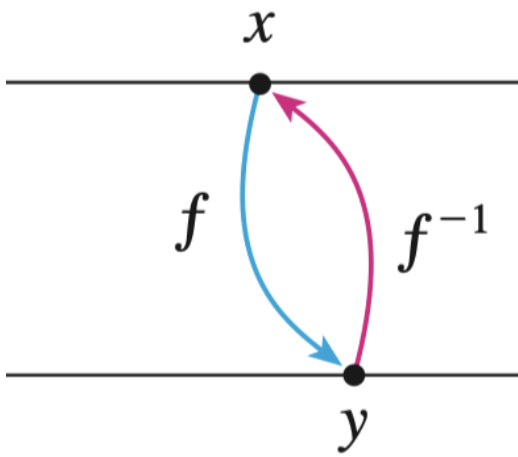


🤔 Domain and range of f and f^{-1} ?

Verifying Inverse Functions

Two functions f and g are inverses of each other if

$$f\Big(g(x)\Big) = x \quad \text{and} \quad g\Big(f(x)\Big) = x$$



Show that $f(x) = 2x^3 - 1$ and $g(x) = \sqrt[3]{\dfrac{x+1}{2}}$ are inverses of each other.

Find the inverse function algebraically.

Example

1 Find the inverse function of $f(x) = \sqrt{2x - 3}$.

Practice

2 If $f(x) = \dfrac{x}{x + 1}$, find $f^{-1}(x)$.

3 If $f(x) = x^5 - 1$, find $f^{-1}(x)$.

Continuity and Differentiability of Inverse Functions

Let f be a function whose domain is an interval I. If f has an inverse function, then the following statements are true.

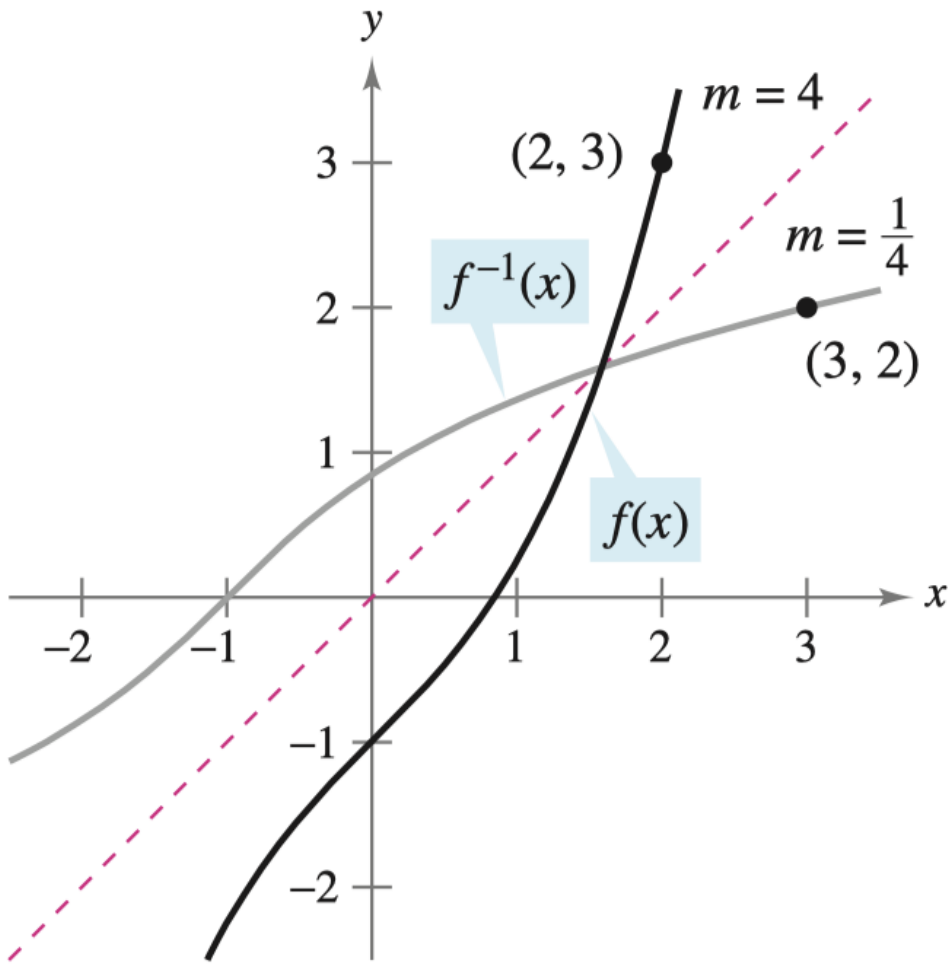
1. If f is continuous on its domain, then f^{-1} is continuous on its domain.
2. If f is increasing on its domain, then f^{-1} is increasing on its domain.
3. If f is decreasing on its domain, then f^{-1} is decreasing on its domain.
4. If f is differentiable at c and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$.

Derivative of the Inverse Function

$$g'(x) = \frac{1}{f'\Big(g(x)\Big)}, \quad f'(g(x) \neq 0$$

Let $f(x) = \frac{1}{4}x^3 + x - 1$.

Find $(f^{-1})'(3)$.



Practice

1 $f(x) = 2x^3 + 3x^2 + 7x + 4$. **Find** $(f^{-1})'(4)$.

2 $f(x) = \sqrt{x^3 + x^2 + x + 1}$. **Find** $(f^{-1})'(2)$.

3 **Multiple Choice**

Let f and g be functions that are differentiable everywhere. If g is the inverse of f and $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

A. 2 B. $\frac{1}{2}$ C. $\frac{1}{5}$ D. $-\frac{1}{2}$ E. -2