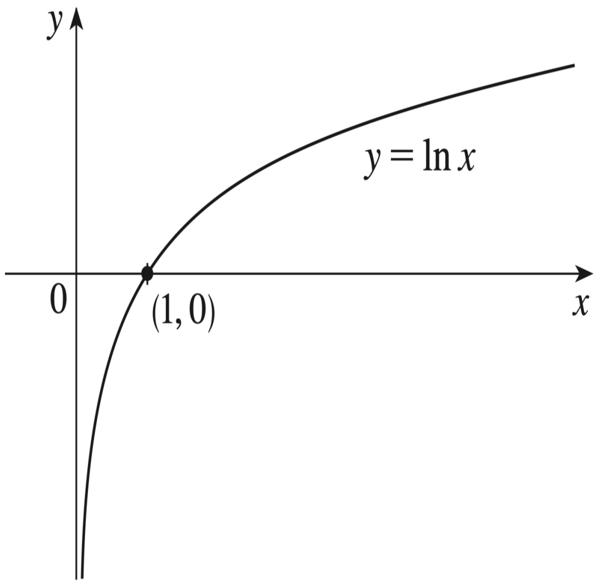
What are the properties of the natural exponential function?

Quick Check

Observe the sketch of $y = \ln(x)$.



1 What are the domain and the range of $y = \ln(x)$?

2 Does $\ln(x)$ have an inverse? If yes, provide a reason and sketch the graph of the inverse function.

3 Evaluate the following limits.

a. $\lim_{x \to 0^+} \ln(x)$ b. $\lim_{x \to \infty} \ln(x)$

Inverse of $\ln(x)$

The inverse function of the natural logarithmic function $f(x) = \ln(x)$ is called the natural exponential function and is denoted by

$$f^{-1}(x) = e^x$$

That is, $y = e^x$ if and only if $x = \ln(y)$

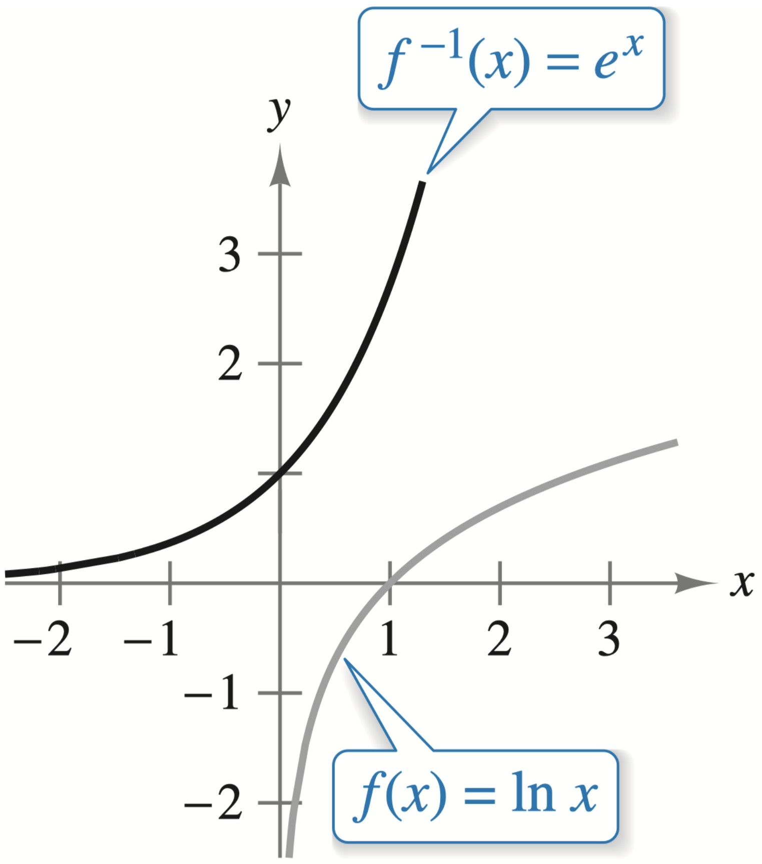
Properties of the Natural Exponential Function

1. The domain of $f(x) = e^x$ is $(-\infty, \infty)$ and the range is $(0, \infty)$.

2. The function $f(x) = e^x$ is continuous, increasing, and one-to-one on its entire domain.

3. The graph of $f(x) = e^x$ is convace upward on its entire domain.

4. $\lim_{x \to -\infty} e^x = 0$ and $\lim_{x \to \infty} e^x = \infty$



Review and Practice

Solve the following exponential equations.

1. $9 - 2e^x = 7$
2. $\ln(x-2)^2 = 12$
3. $-6 + 3e^x = 8$

Derivatice of the natural exponential function

Consider $y = e^x$ if and only if $x = \ln(y)$. Then,

Let u be a differentiable function of x.

$$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$$

Practice

1. $\dfrac{d}{dx}\left[e^{2x-1}\right]$
2. $\dfrac{d}{dx}\left[e^{-3/x}\right]$
3. $\dfrac{d}{dx}\left[e^{x}\ln x\right]$

Find the derivative.

4. $y = x^3 e^x$
5. $y = \ln(1 + e^{2x})$
6. $y = e^x(\sin(x) + \cos(x))$
7. Find the equation of the tangent line to the graph of $f(x) = xe^x - e^x$ at the point $(1, 0)$.

Integral of e^x

Because the exponential function $y = e^x$ has a simple derivative, its integral is also simple.

Example

1. $\int x^2 e^{x^3}\, dx$

2. Find the area under the curve $y = e^{-3x}$ from 0 to 1.

Practice

1. $$\int e^{2x-1}\,dx$$
2. $$\int \frac{e^{1/x}}{x^2}\,dx$$
3. $$\int e^x\sqrt{1-e^x}\,dx$$
4. $$\int 5xe^{-x^2}\,dx$$
5. $$\int \sin x \cdot e^{\cos x}\,dx$$
6. $$\int \frac{5-e^x}{e^{2x}}\,dx$$
7. $$\int_0^1 \frac{e^x}{1+e^x}\,dx$$