

What methods can be used to differentiate and integrate exponential functions with bases other than  $e$ ?

## Quick Check

Without integrating, state the integration formula/ steps that you would take to integrate each of the following.

$$1 \int \frac{e^x}{e^x + 1} dx$$

$$2 \int xe^{x^2} dx$$

## Exponential functions to Base $a$

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If  $a$  is a positive real number ( $a \neq 1$ ) and  $x$  is any real number, then the exponential function to the base  $a$  is denoted  $a^x$  and is defined by

$$a^x = e^{(\ln a) \cdot x}$$

If  $a = 1$ , then  $1^x$  is a constant function.

Some familiar properties:

$$1. a^0 = 1$$

$$2. a^x a^y = a^{x+y}$$

$$3. \frac{a^x}{a^y} = a^{x-y}$$

$$4. (a^x)^y = a^{xy}$$

# Evaluating Logarithms

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$$a^x = y \iff \log_a y = x$$

1  $\log_{10} 1000$

2  $\log_2 32$

3  $\log_{10} .1$

1  $\log_{16} 4$

## Logarithm function to Base $a$

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If  $a$  is a positive real number ( $a \neq 1$ ) and  $x$  is any real number, then the logarithmic function to the base  $a$  is denoted  $\log_a x$  and is defined by

$$\log_a x = \frac{1}{\ln a} \ln x$$

Some familiar properties:

$$1. \log_a 1 = 0$$

$$2. \log_a xy = \log_a x + \log_a y$$

$$3. \log_a x^n = n \log_a x$$

$$4. \log_a \frac{x}{y} = \log_a x - \log_a y$$

## Quick Review

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**1**  $3^x = \frac{1}{81}$

**2**  $\log_2 x = -4$

**3**  $\log_2 (x - 1) = 5$

# Derivatives of Exponential and Logarithmic Functions

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Let  $a$  be a real number ( $a \neq 1$ ) and  $u$  be a differentiable function of  $x$ .

$$a^x = e^{(\ln a)x}$$

$$1. \frac{d}{dx} a^x =$$

$$2. \frac{d}{dx} a^u =$$

$$\log_a x = \frac{1}{\ln a} \ln x$$

$$3. \frac{d}{dx} \log_a x =$$

$$4. \frac{d}{dx} \log_a u =$$

# Practice

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Differentiate each function.

## Example

1.  $y = 2^x$

2.  $y = 2^{x^2+3x}$

3.  $y = \log_{10} \cos x$

4.  $y = x^{2/x}$

## Practice

1  $f(x) = 4^x$

2  $g(t) = t^2 2^t$

3  $f(x) = \log_2 \frac{x^2}{x-1}$

4  $f(x) = (\ln x)^{\cos x}$

! careful!

5  $y = e^e$

6  $y = e^x$

7  $y = x^e$

8  $y = x^x$

# Integrating Exponential Functions to another base

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$$a^x = e^{(\ln a)x}$$

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$$\int a^x dx =$$

Example:

$$\int 2^x dx =$$



# Practice

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1  $\int 5^{-x} dx$

2  $\int (3 - x)7^{(3-x)^2} dx$

3  $\int 2^{\sin x} \cdot \cos x dx$

4  $\int_1^e 6^x - 2^x dx$