What are separable differentiable equations?

Quick Check

Determine whether the given value is a solution to the equation.

$$\frac{1}{x} - \frac{1}{x-4} = 1$$

1 $x = 2$

2 $x = 4$

Differential Equation

A differential equation is an equation that contains an unknown function and one or more of its derivatives.

Examples of differential equations:

1. $\dfrac{dy}{dx} = \dfrac{2y}{x}$
2. $xy' = y^2 \ln x$
3. $\dfrac{dy}{dt} = 1 + t + y + ty$
4. $y'' + 2y' = 2e^x$
5. $yy' - 2x = 0$

Solution

A function $f(x)$ is called a solution of a differential equation if the equation is satisfied when y and its derivatives are replaced by $f(x)$ and its derivatives.

Example

Show that $y = e^{-2x}$ is a solution of the differential equation $y' + 2y = 0$.

Verifying Solutions

1 Determine whether the function is a solution of the differential equation $y'' - y = 0$.

a. $y = \sin x$ b. $y = 4e^{-x}$ c. $y = Ce^{x}$

2 Is $y = Ce^{-x/2}$ a solution to the differential equation $2y' + y = 0$?

General Solution

Find the general solution of the differential equation $\frac{dy}{dx} = \cos(2x)$.

Separable Differential Equations

Equations that can be written in the form $M(x) + N(y)\dfrac{dy}{dx} = 0$ can be solved via separation of variables.

Examples

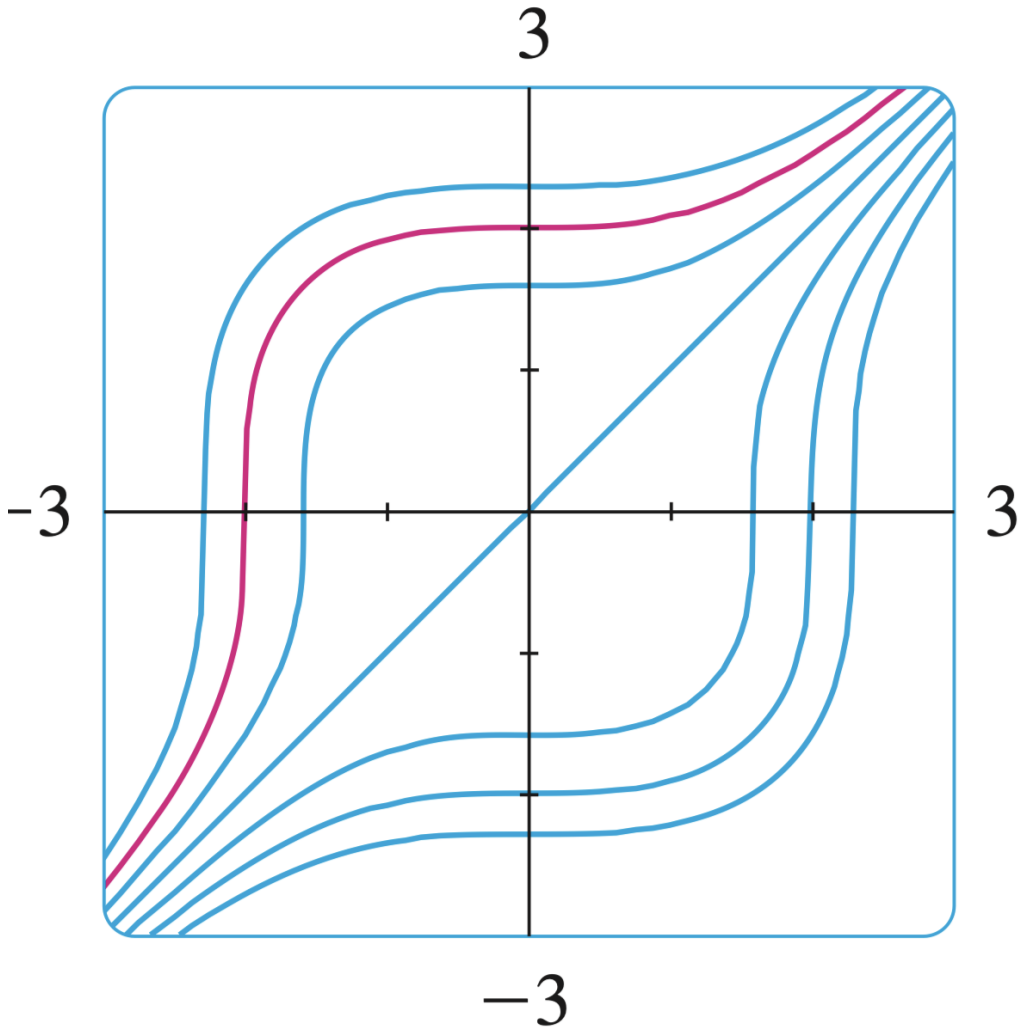
$$x^2 + 3y\frac{dy}{dx} = 0 \quad \rightarrow \quad 3y\,dy = -x^2\,dx$$

$$\frac{xy'}{e^y + 1} = 2 \quad \rightarrow \quad \frac{1}{e^y + 1}dy = \frac{2}{x}dx$$

Particular Solution

1 Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$.

2 Find the solution of this differential equation that satisfies the initial condition $y(0) = 2$.



Examples

1 Solve the differntial equation $\dfrac{dy}{dx} = \dfrac{6x^2}{2y + \cos y}$.

2 Solve the equation $y' = x^2 y$. Write the solution in the form $y = f(x)$.

Practice

Solve the differential equation.

1. $\dfrac{dy}{dx} = \dfrac{y}{x}$
2. $\dfrac{dy}{dx} = \dfrac{\sqrt{x}}{e^y}$
3. $y' = y^2 \sin x$
4. $(x^2 + 1)y' = xy$

Find the solution that satisfies the given initial condition.

5. $\sqrt{x} + \sqrt{y}y' = 0 \qquad y(1) = 4$
6. $y\sqrt{1 - x^2}y' - x\sqrt{1 - y^2} = 0 \qquad y(0) = 1$