How are exponential functions used to model growth and decay in applied problems?

Quick Check

Solve the differential equation
$$y' = \frac{2x}{y}$$
.

In many applications the rate of change of the variable y is proportional to the value of y itself. If y is a function of time t, the proportion can be written as follows.

$$rac{dy}{dt} = ky$$

Rate of change of y is proportional to y

Example of an exponential growth model

1 The rate of change of y is proportional to y. When t = 0, y = 2. When t = 2, y = 4. What is the value of y when t = 3?

Radioactive Decay

2 Suppose that 10 grams of plutonium isotope-239 was released in the Chernobyl nuclear accident. How long will it take 10g to decay to 1g?

3 Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Aprroximately how many flies were in the original population.

A Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

a.
$$\frac{3\ln 3}{\ln 2}$$
 b. $\frac{2\ln 3}{\ln 2}$ c. $\frac{\ln 3}{\ln 2}$ d. $\ln \left(\frac{27}{2}\right)$ e. $\ln \left(\frac{9}{2}\right)$

5 During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the pandemic is first discovered?

a. 343	b. 1.343	c. 1.367	d. 1.400	e. 2.057

A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

a. 4.2 pounds b. 4.6 pounds c. 4.8 pounds d. 5.6 pounds e. 6.5 pounds

7 The rate of change of volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

a.
$$V(t) = k\sqrt{t}$$
 b. $V(t) = k\sqrt{V}$ c. $\frac{dV}{dt} = k\sqrt{t}$ d. $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$ e. $\frac{dV}{dt} = k\sqrt{V}$

8 A pizza, heated to a temperature of 350 degrees Farhenheit, is taken out of an oven and placed in a 75^0F room at t = 0 minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Farhenheit per minute. To the nearest degree, what is the temperature of the pizza at time t = 5 minutes?

Logistic Growth

When describing a population, there often exists some upper limit L past which growth cannot occur. (k and L are positive constants.)

$$egin{aligned} rac{dy}{dt} &= kyigg(1-rac{y}{L}igg) \ &\downarrow \ &y &= rac{L}{1+be^{-kt}} \end{aligned}$$



Note that as $t \to \infty$, $y \to L$.

A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population p is

$$\frac{dp}{dt} = kp \left(1 - \frac{p}{4000} \right), \quad 40 \le p \le 4000$$

where *t* is the number of years.

- **a.** Write a model for the elk population in terms of *t*.
- **b.** Graph the slope field of the differential equation and the solution that passes through the point (0, 40).
- c. Use the model to estimate the elk population after 15 years.
- **d.** Find the limit of the model as $t \rightarrow \infty$.