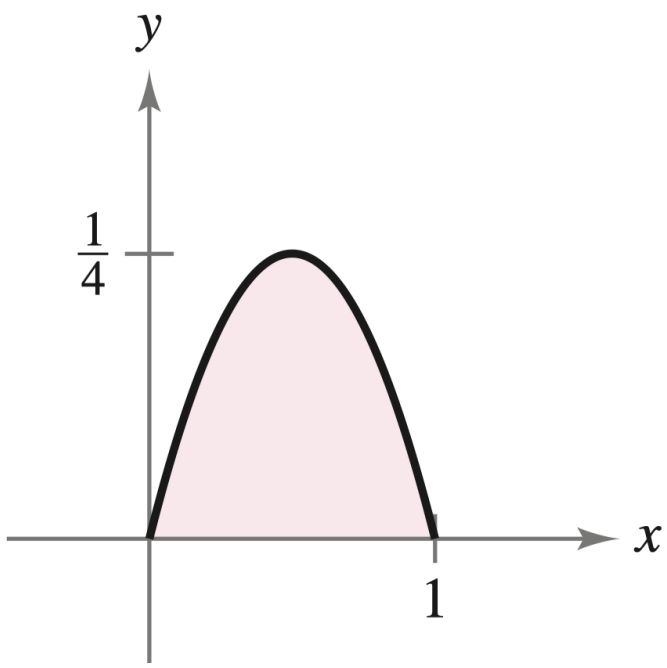
How is the definite integral used to compute areas between curves?

Quick Check

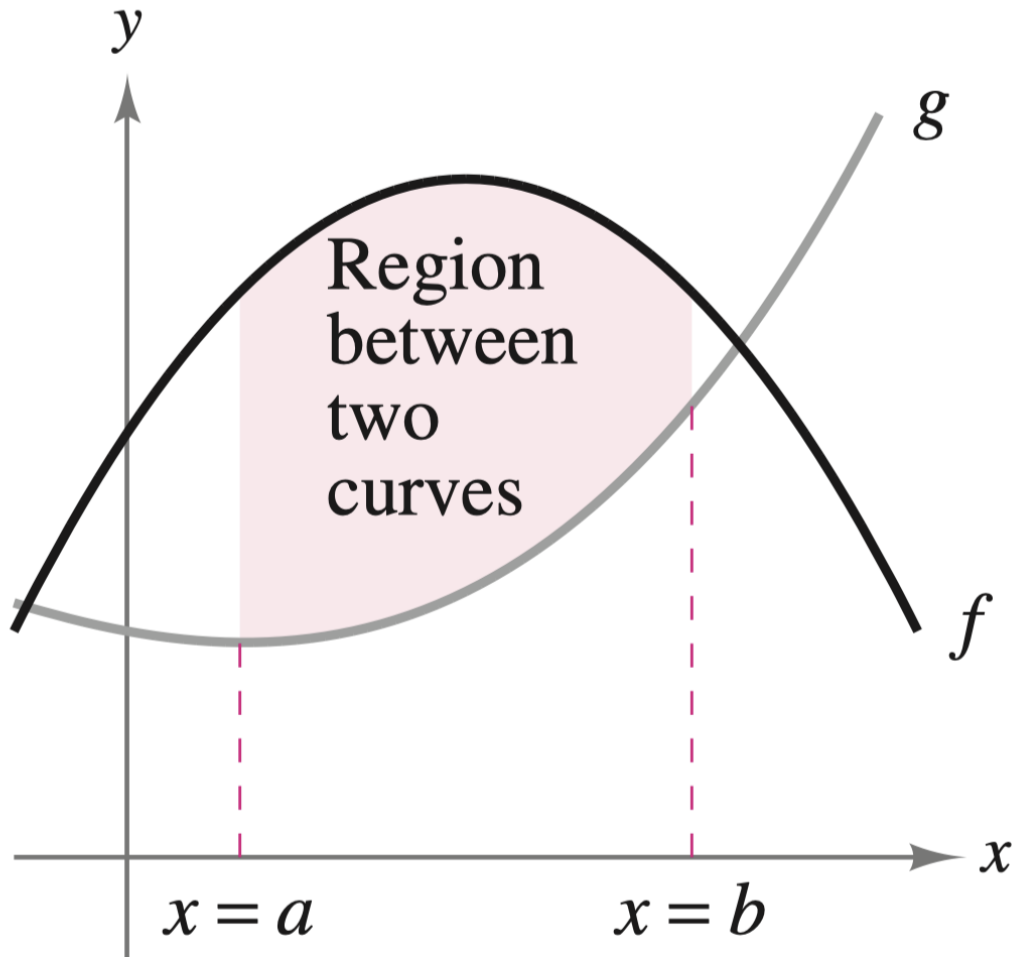
Find the area of the shaded region.

$$y = x - x^2$$

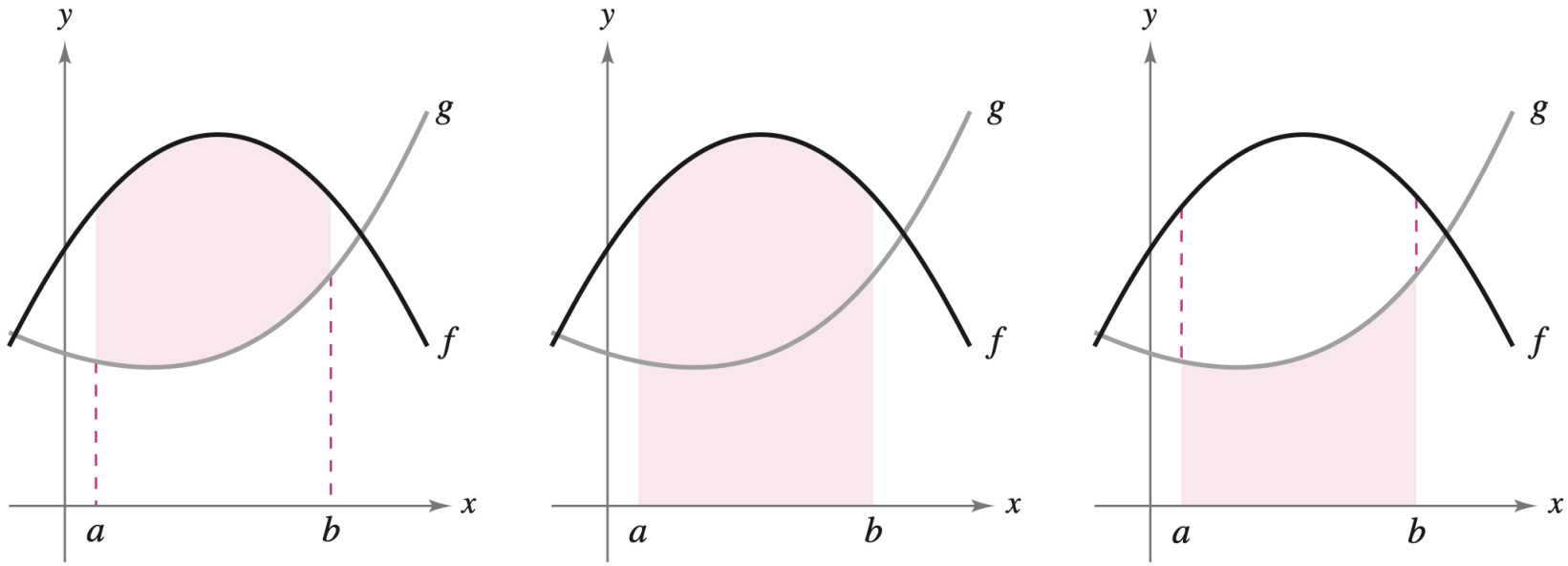


Area between two curves

Explain how you could find the area of the highlighted region between the two curves shown below.

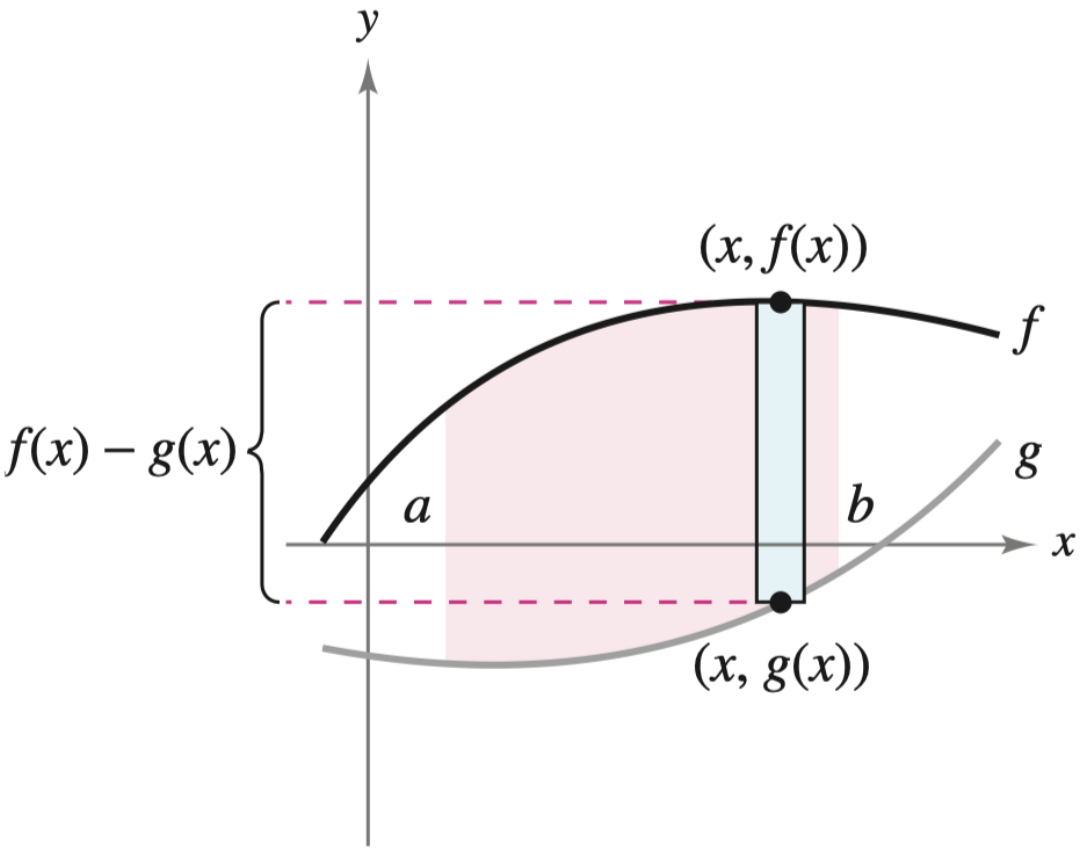


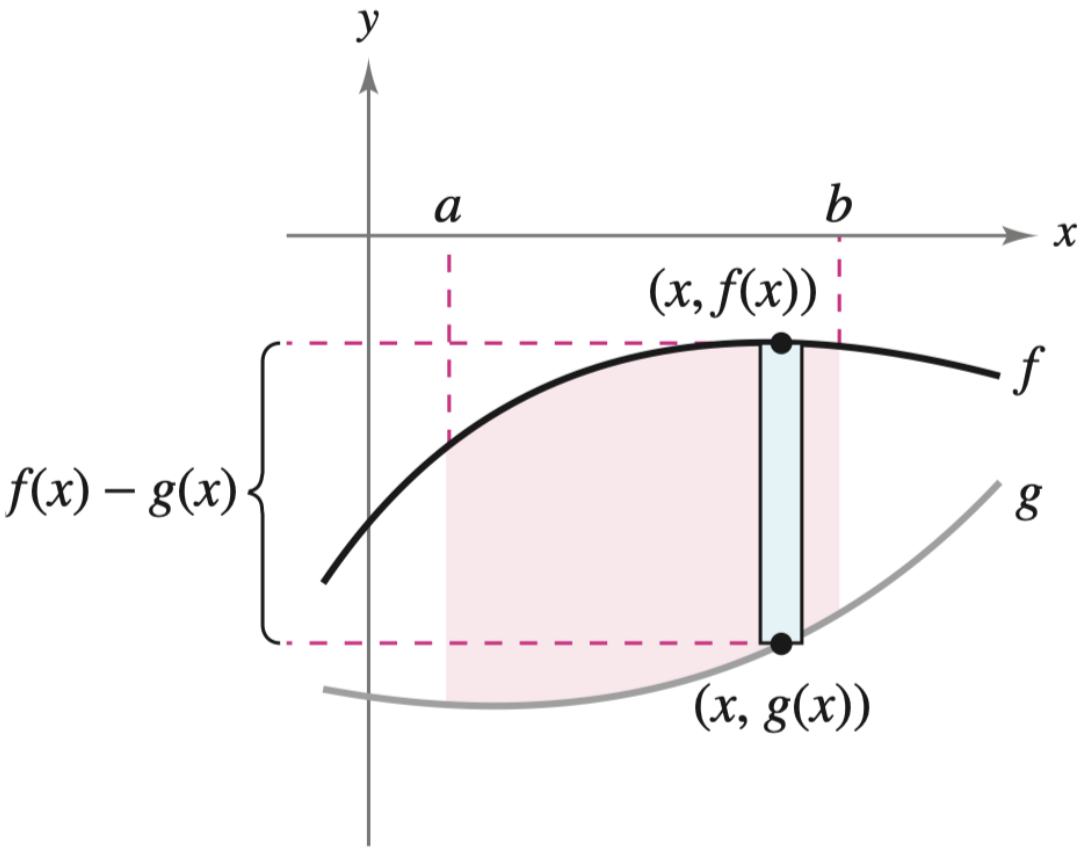
Idea



Area of region between f and g	$=$	Area of region under f	$-$	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	$=$	$\int_a^b f(x)\,dx$	$-$	$\int_a^b g(x)\,dx$

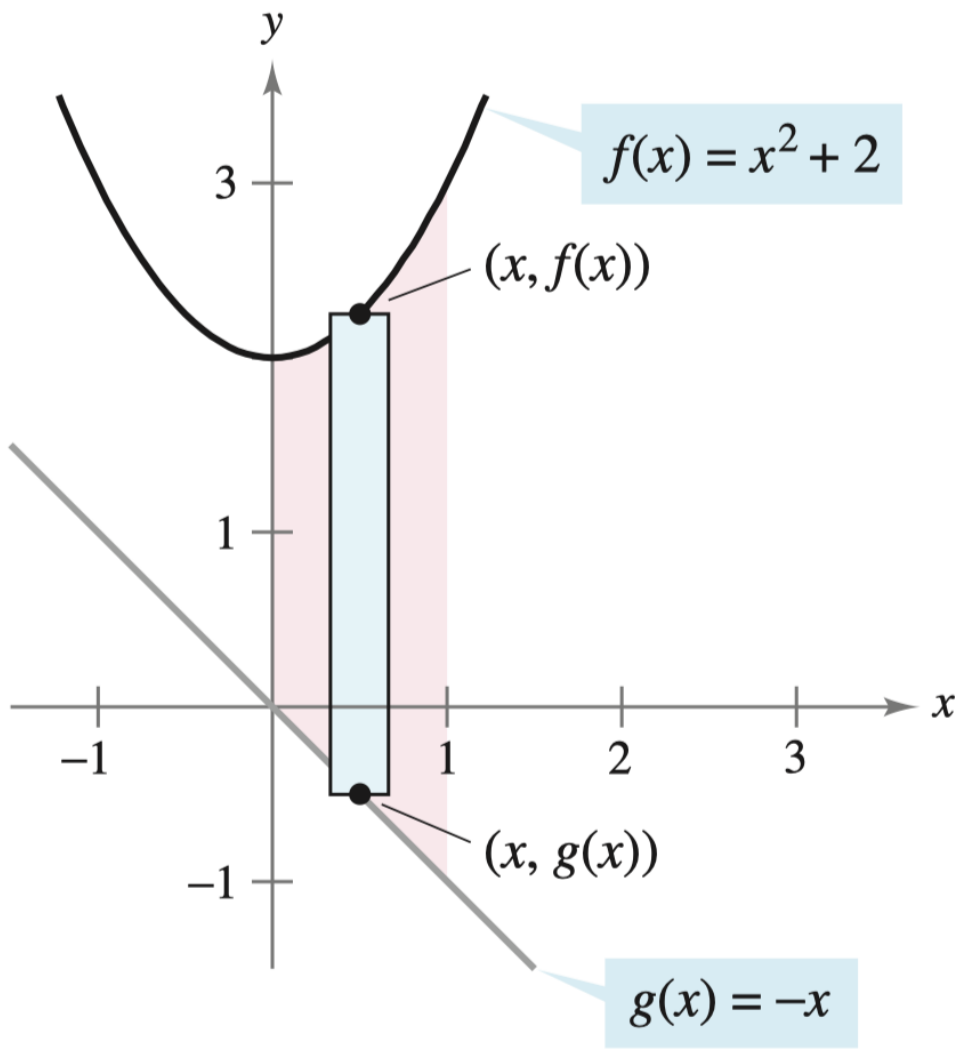
Idea





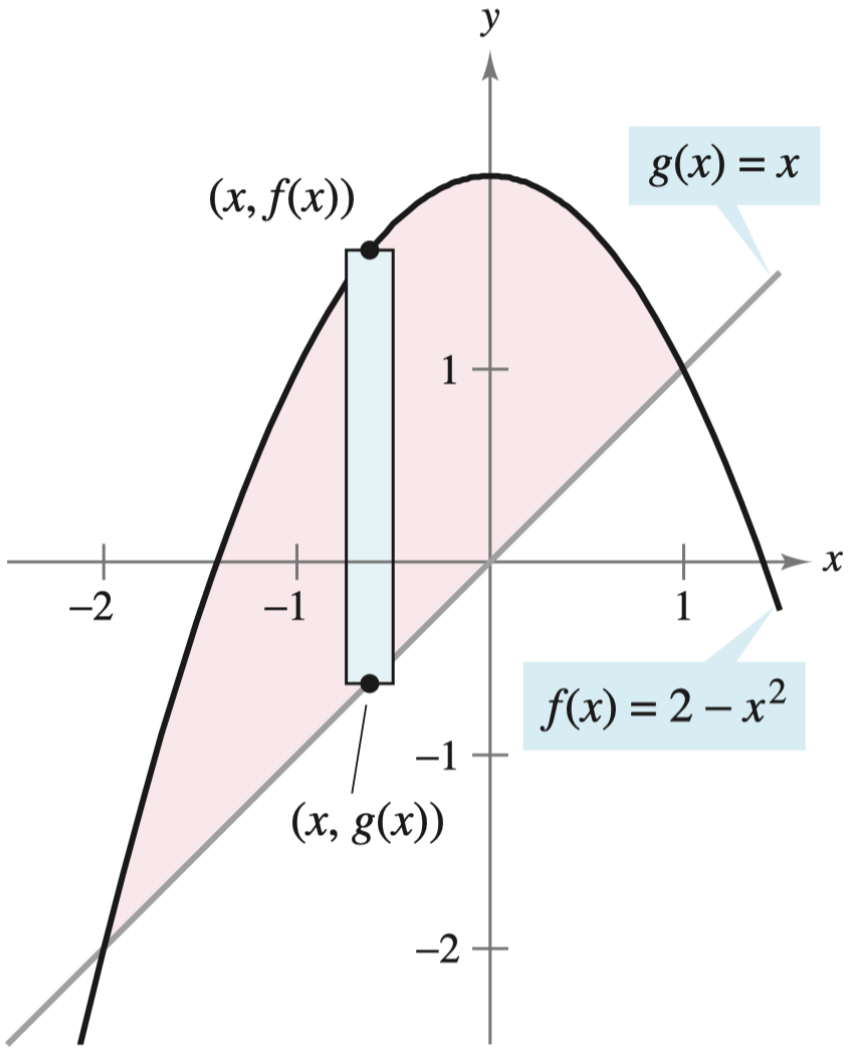
Finding the Area of a Region Between Two Curves

Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.



A Region Lying Between Two Intersecting Graphs

Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.

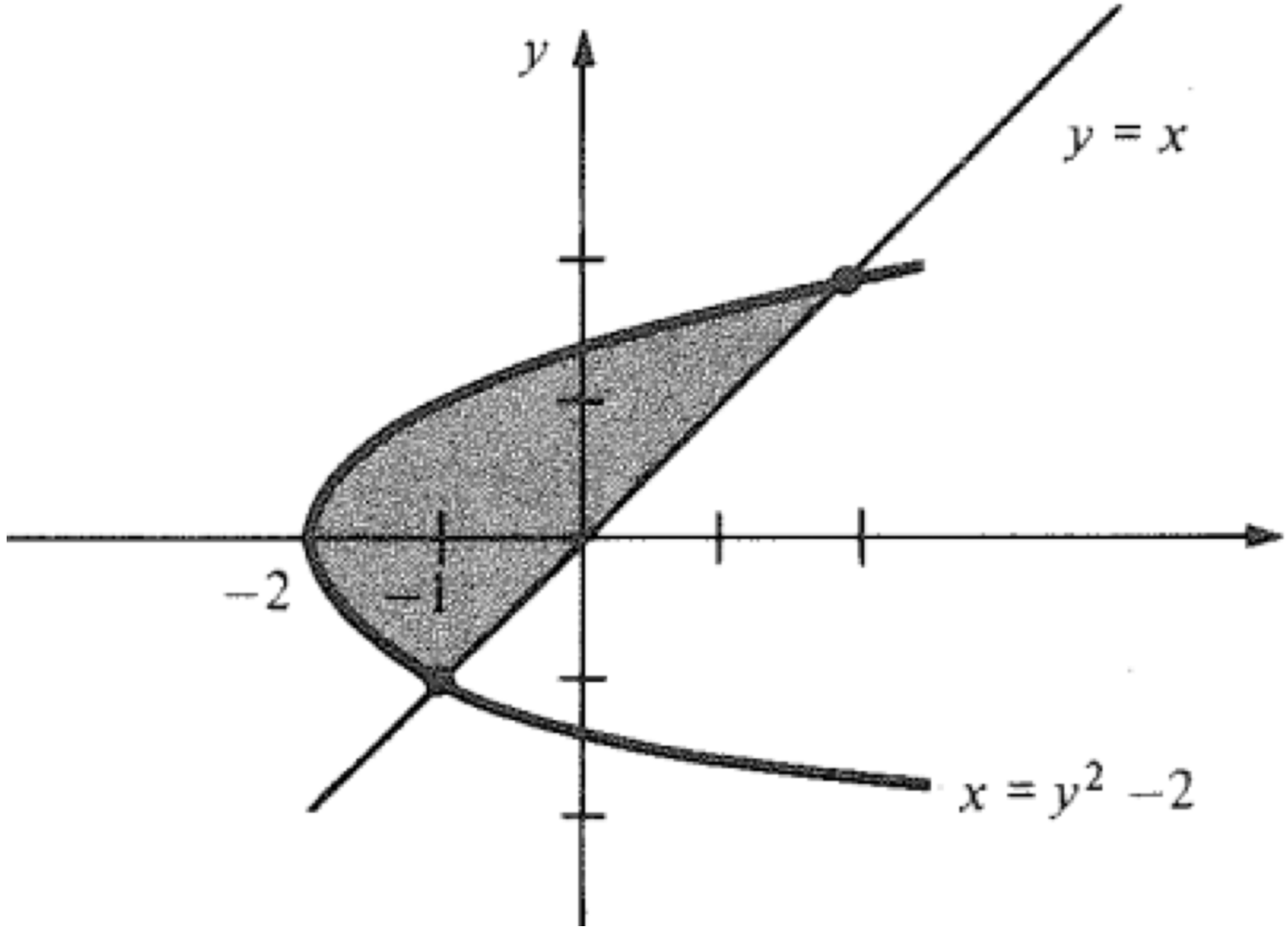


Curves That Intersect at More Than Two Points

Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

With respect 🙏 to y

Find the area between the graphs of $x = y^2 - 2$ and $y = x$.



Summary (Continue with practice on handout)

To find areas between curves

↓

$$A = \int_{x_1}^{x_2} \underbrace{[(\text{top curve}) - (\text{bottom curve})]}_{\text{in variable } x}\, dx$$ Vertical rectangles

$$A = \int_{y_1}^{y_2} \underbrace{[(\text{right curve}) - (\text{left curve})]}_{\text{in variable } y}\, dy$$ Horizontal rectangles