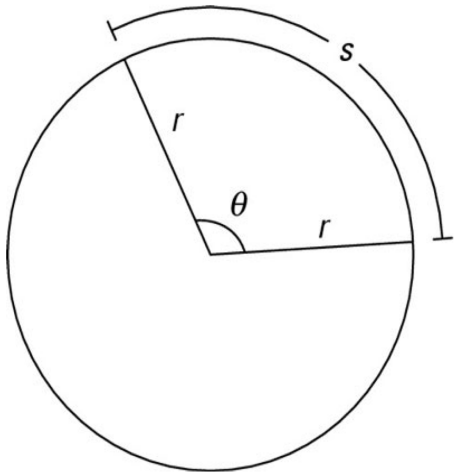
What do we mean by the length of a curve and how do we measure it?

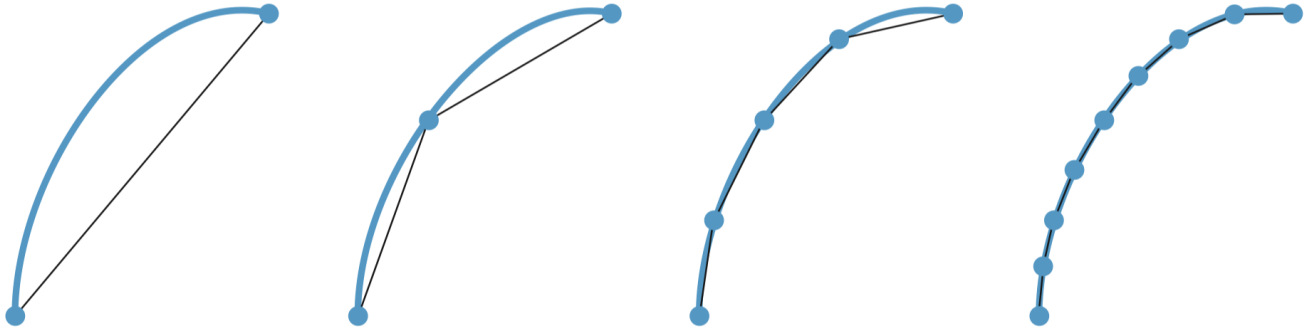
Quick Check

1. Find the length of the line segment formed by joining the points (1,1) and (3,4).
2. Look at the image below. What connection(s) can you surmise between r, θ, and s?



Length of a Curve

🤔 How is the distance formula used?

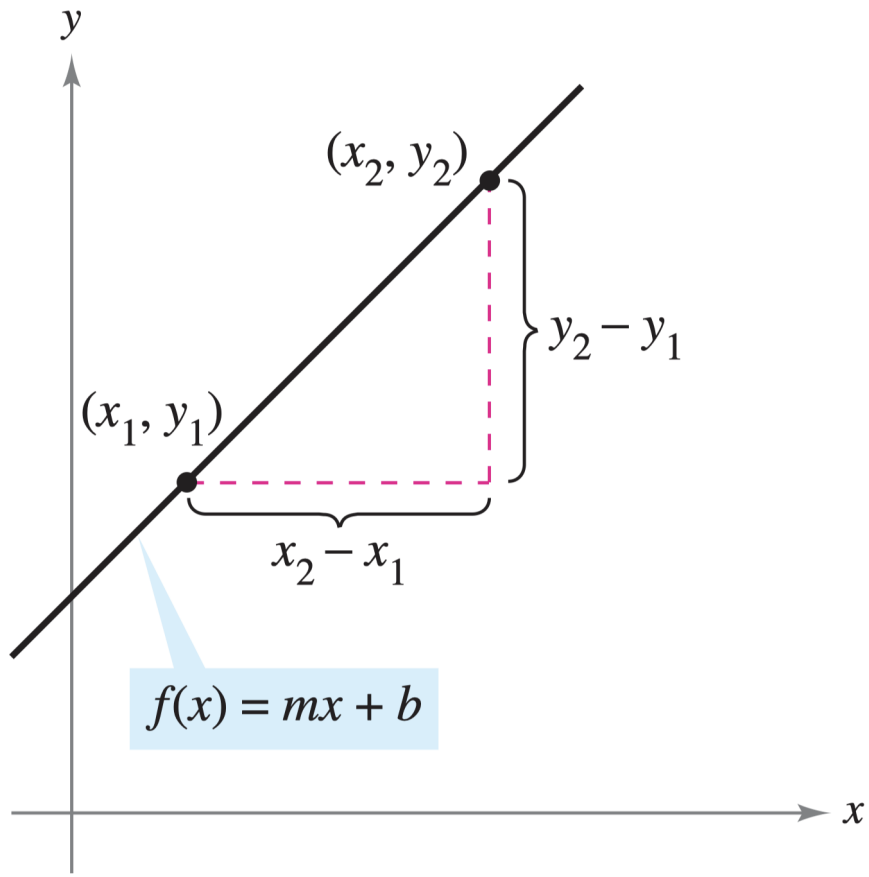


Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The arclength of f between a and b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

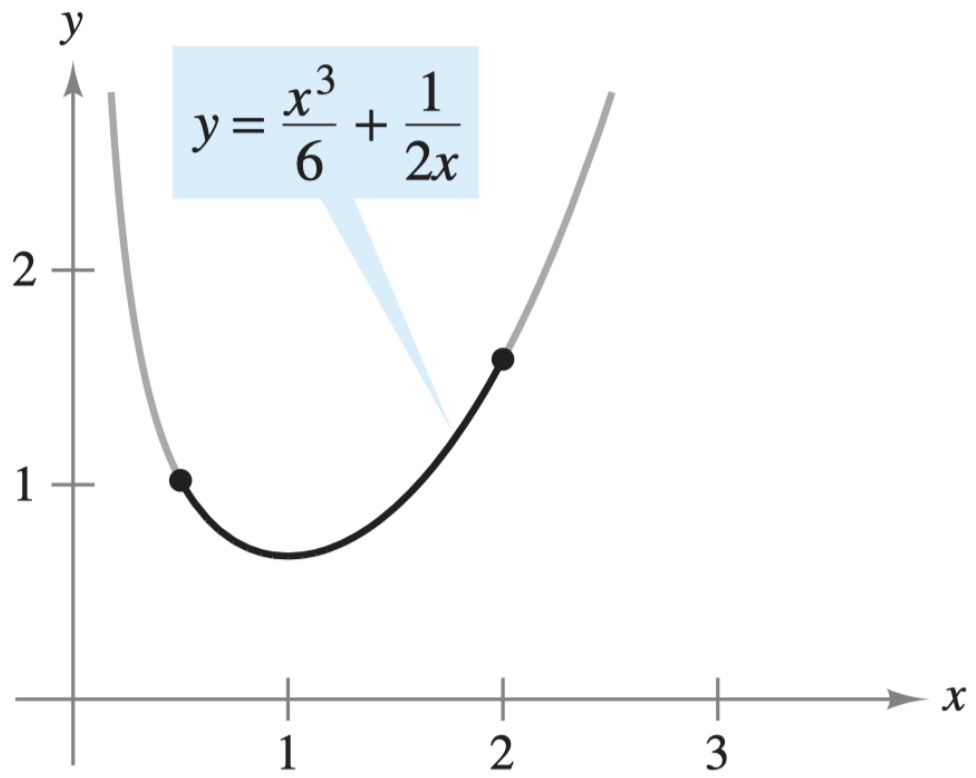
A Simple Case for a Test

Find the arclength for (x_1, y_1) to (x_2, y_2) on the graph of a linear function $y = mx + b$.



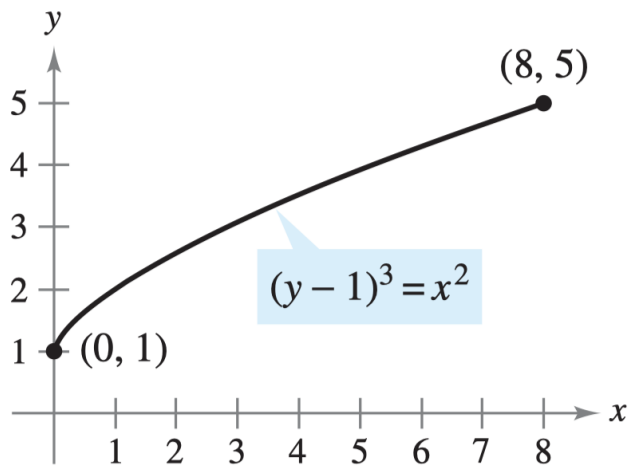
Arc Length

Find the arclength of the graph of $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $\left[\frac{1}{2}, 2\right]$.



Practice

1 Find the arclength of the graph of $(y-1)^3 = x^2$ on the interval $[0, 8]$.



2 Find the arclength of the graph of $y = \frac{2}{3}x^{3/2} + 1$ on the interval $[0, 8]$.

3 Find the arclength of the graph of $x = \frac{1}{3}(y^2+2)^{3/2}$ on the interval $0 \leq y \leq 4$.

Distance Travelled

The graph of the equation $y = \frac{1}{4}x^{3/2}$ gives the course taken by an oil tanker after leaving port, which is taken to be located at the origin of a coordinate system. Find the distance traveled by the tanker when it reaches a point on the course that is located 4 mi to the east and 2 mi to the north of the port.