How do you determine which integration rule to use in a given situation?

Quick Check

Find each antiderivative.

1 $\int \frac{4}{x^2+9}\,dx$

2 $\int \frac{4x}{x^2+9}\,dx$

3 $\int \frac{4x^2}{x^2+9}\,dx$

Integration Strategies

1 Breakup

$$\int \frac{x+3}{\sqrt{4-x^2}}\,dx$$

2 Re-write as squares

$$\int \frac{x^2}{\sqrt{16-x^6}}\,dx$$

3 Disguised LOG

$$\int \frac{1}{1+e^x}\,dx$$

4 Disguised POWER Rule

$$\int (\cot x)[\ln(\sin x)]\,dx$$

5 TRIG Identities

$$\int \tan^2(2x)\,dx$$

Simplification Strategies for Integrals

Technique	Example
Expand (numerator).	$(1 + e^x)^2 = 1 + 2e^x + e^{2x}$
Separate numerator.	$\dfrac{1 + x}{x^2 + 1} = \dfrac{1}{x^2 + 1} + \dfrac{x}{x^2 + 1}$
Complete the square.	$\dfrac{1}{\sqrt{2x - x^2}} = \dfrac{1}{\sqrt{1 - (x - 1)^2}}$
Divide improper rational function.	$\dfrac{x^2}{x^2 + 1} = 1 - \dfrac{1}{x^2 + 1}$
Add and subtract terms in numerator.	$\dfrac{2x}{x^2 + 2x + 1} = \dfrac{2x + 2 - 2}{x^2 + 2x + 1}$ $= \dfrac{2x + 2}{x^2 + 2x + 1} - \dfrac{2}{(x + 1)^2}$
Use trigonometric identities.	$\cot^2 x = \csc^2 x - 1$
Multiply and divide by Pythagorean conjugate.	$\dfrac{1}{1 + \sin x} = \left(\dfrac{1}{1 + \sin x}\right)\left(\dfrac{1 - \sin x}{1 - \sin x}\right)$

Practice

1. $\int \frac{9}{(t-8)^2}\,dt$
2. $\int t^2\sqrt[3]{t^3-1}\,dt$
3. $\int x - \frac{5}{(3x+5)^2}\,dx$
4. $\int \frac{1}{(7x-2)} - \frac{1}{(7x+2)}\,dx$
5. $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$
6. $\int \frac{1+\cos x}{\sin x}\,dx$
7. $\int \frac{e^{1/t}}{t^2}\,dt$
8. $\int (\tan x)[\ln(\cos x)]\,dx$
9. $\int \frac{1}{9+5x^2}\,dx$