What connection is there between integration by parts and the product rule for differentiation?

Quick Check

State the integration formula you would use to perform the integration. DO NOT integrate!

1. $\int x(x^2+1)^3\,dx$
2. $\int \frac{1}{x^2+1}\,dx$
3. $\int x\sec(x^2+1)\tan(x^2+1)\,dx$

Integrals involving products 🤔

Test whether $\int f(x)g(x)dx = \int f(x)dx \cdot \int g(x)dx$ is true or false.

How can we deal with ↓

$$\int x \ln x \, dx \qquad \int e^x \sin x \, dx \qquad \int x^2 e^x \, dx \qquad \int \ln x \, dx$$

Recall the product rule for derivatives and the definition of the antiderivative.

Let u and v be differentiable functions of x, then

$$\frac{d}{dx}[uv] =$$

Integration by Parts $\int u\,dv = uv - \int v\,du$

Find $\int xe^x\,dx$

$$\int \underbrace{(x)}_{u}\underbrace{(e^x dx)}_{dv}, \quad \int \underbrace{(e^x)}_{u}\underbrace{(x\,dx)}_{dv}, \quad \int \underbrace{(1)}_{u}\underbrace{(xe^x\,dx)}_{dv}, \quad \int \underbrace{(xe^x)}_{u}\underbrace{(dx)}_{dv}$$

Examples

1. $\int x^2 \ln x \, dx$

2. $\int \ln x \, dx$

3. $\int \arcsin x \, dx$

Repeated use of Integration by Parts

$$\int e^x \sin x \, dx$$

Table Method

$$\int x^2 \sin x \, dx$$

Signs	u	dv
+	_____	_____
-		
+		
-		

Thinking about Integration by Parts

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \quad \int x^n \sin ax\,dx, \quad \text{or} \quad \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}\,dx$, $\sin ax\,dx$, or $\cos ax\,dx$.

2. For integrals of the form

$$\int x^n \ln x\,dx, \quad \int x^n \arcsin ax\,dx, \quad \text{or} \quad \int x^n \arctan ax\,dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\,dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \quad \text{or} \quad \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\,dx$.

Practice

1. $\int xe^{-2x}\,dx$
2. $\int x^2e^{x^3}\,dx$
3. $\int \frac{(\ln x)^2}{x}\,dx$
4. $\int x^3\sin x\,dx$
5. $\int e^{4x}\cos x\,dx$
6. $\int x^4\ln x\,dx$