How do we deal with limits that produce indeterminate forms?

Quick Check

Evaluate each limit.

$$\lim_{x \to -1} \frac{2x^2 - 2}{x + 1} \qquad \lim_{x \to \infty} \frac{3x^2 - 1}{2x^2 + 1} \qquad \lim_{x \to 0} \frac{e^{2x} - 1}{e^x - 1} \qquad \lim_{x \to 0} \frac{\sqrt{4 + x} - 2}{x}$$

Indeterminate Forms

🔳 Not all limits involving indeterminate forms can be evaluated using algebraic manipulation. Consider the following example.

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x}$$

$$\lim_{x \to 0} \frac{e^{2x}}{x} - \frac{1}{x}$$

Maybe use a graph or a table to find the limit??

L'hopital's Rule

Let f and g be differentiable functions on (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of $\dfrac{f(x)}{g(x)}$ as $x \to a$ produces an indeterminate form of type $\dfrac{0}{0}$ or $\pm\dfrac{\infty}{\infty}$,

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided that the limit on the right exists or is infinite.

Applying L'hopital's Rule

Indeterminate form $\frac{0}{0}$.

Evaluate

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x}$$

Applying L'hopital's Rule

1 Indeterminate form $\frac{\infty}{\infty}$.

$$\lim_{x \to \infty} \frac{\ln x}{x}$$

2 Indeterminate form $\frac{\infty}{\infty}$.

$$\lim_{x \to -\infty} \frac{x^2}{e^{-x}}$$

Other indeterminate forms

$0 \cdot \infty$, 1^{∞}, ∞^{0}, 0^{0}, and $\infty - \infty$

Example:

1. $\lim_{x \to \infty} e^{-x}\sqrt{x}$
2. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$

Not Indeterminate

$$\infty + \infty \to \infty$$

$$-\infty - \infty \to -\infty$$

$$0^{\infty} \to 0$$

$$0^{-\infty} \to \infty$$

🤔 What's Wrong?

$$\lim_{x \to 0} \frac{e^x}{x} = \frac{e^x}{1} = 1$$

Practice

1. $\lim_{x \to \infty} \frac{x^2}{e^x}$
2. $\lim_{x \to \infty} \frac{\ln x^4}{x^3}$
3. $\lim_{x \to 0} \frac{\sin^2(3x)}{4x}$
4. $\lim_{x \to 0} \frac{\sin x}{x}$
5. $\lim_{x \to 0^+} (\sin x)^x$
6. $\lim_{x \to 0} \frac{1 - \cos x}{x}$
7. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$