What is the Integral Test and how is it applied?

Quick Check - True / False. Explain.

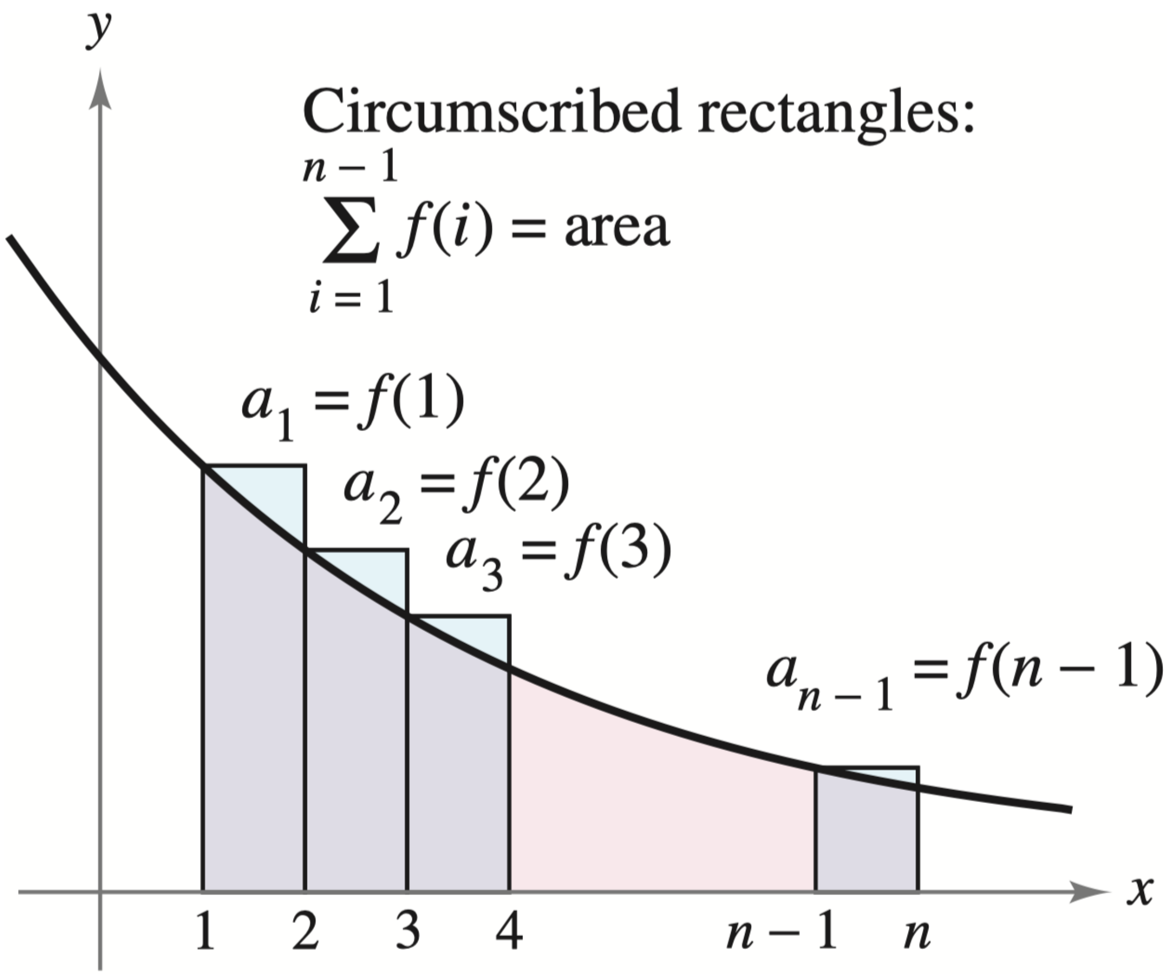
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

The Integral Test

If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \text{ and } \int_1^{\infty} f(x)\,dx$$

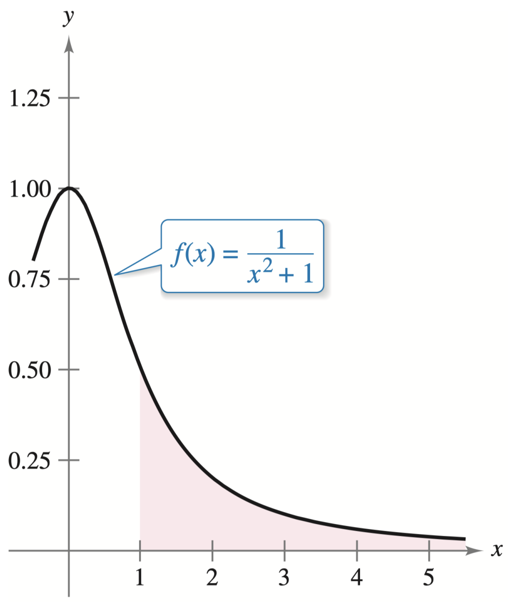
either both converge or diverge.



Check conditions and use the Integral Test

1 Apply the Integral Test to the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

2 Apply the Integral Test to the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$



$p-$series and Harmonic Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots \quad \text{p-series}$$

Special case where $p = 1$

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \quad \text{Harmonic Series}$$

Discuss how Harmonic series can be shown to diverge.

Convergence of $p-$series

The $p-$series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$$

1. converges if $p > 1$

2. diverges if $0 < p \leq 1$

Proof by Integral Test - Break into several cases: $\rightarrow p = 1, p > 1, 0 < p \leq 1$

$$\int_1^{\infty} \frac{1}{x^p} dx$$

Convergent and Divergent $p-$series

1. Does a $p-$series with $p = 2$ converge or diverge.

2. Does the following series converge or diverge. Use any method.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$$

Practice

Use the integral test to determine the convergence or divergence of the series.

1. $$\sum_{n=1}^{\infty} \frac{2}{3n+5}$$

2. $$\sum_{n=1}^{\infty} ne^{(-n/2)}$$

3. $$\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$$

4. $$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$

5. $$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

6. $$\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$$