How can we use comparison of series to determine whether a series converges or diverges??

Quick Check

Determine the convergence or divergence of the series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}}$

3. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

5. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{\sqrt{n}}\right)^n$

6. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

Comparison of Series

The second series cannot be tested by the same convergence test as the first series even though it is similar to the first.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$ is geometric, but $\displaystyle\sum_{n=0}^{\infty} \frac{n}{2^n}$ is NOT.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a $p-$series, but $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ is NOT.

3. $\displaystyle a_n = \frac{n}{(n^2+3)^2}$ is easily integrated, but $\displaystyle b_n = \frac{n^2}{(n^2+3)^2}$ is NOT.

Direct Comparison Test

Allows you to compare a **positive termed** series with complicated terms with a simpler series whose convergence or divergence is known.

Let $0 < a_n < b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If the 'larger' series converges, the 'smaller' series must also converge.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

If the 'smaller' series diverges, the 'larger' series must also diverge.

Direct Comparison Test

Determine the convergence or divergence.

1. $$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$$

2. $$\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$$

Limit Comparison Test

Suppose a_n and b_n are positive termed series.

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$$

where L is finite and positive. Then the two series either both converge or both diverge.

Example:

Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{an+b}$ where a and b are positive, converge or diverge.

Practice

Determine the convergence or divergence.

1 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$ compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

2 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$ compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

3 $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$ compare with $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^5} \to \sum_{n=1}^{\infty} \frac{1}{n^3}$.

Practice

Determine the convergence or divergence.

4 $$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

5 $$\sum_{n=1}^{\infty} \frac{n2^n}{4n^3+1}$$