How do we deal with series that don't have just positive terms?

Quick Check

Consider the sequence $a_n = \dfrac{1}{(2n+1)^3}$

a. Find $\lim\limits_{n \to \infty} a_n$

b. Find the first four partial sums for the series $\sum\limits_{n=1}^{\infty} a_n$

c. Use the Integral Test to determine if $\sum\limits_{n=1}^{\infty} a_n$ converges or diverges.

Alternating Series Test

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \text{ and } \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge if the following two conditions are met

1. $\lim_{n \to \infty} a_n = 0$

2. $a_{n+1} \leq a_n$ for all n.

Using the Alternating Series Test

Determine the convergence or divergence of each series.

1. $$\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n = \sum_{n=0}^{\infty}(-1)^n\frac{1}{2^n}$$

write out few terms

2. $$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$$

3. $$\sum_{n=1}^{\infty}\frac{n}{(-2)^{n-1}}$$

Case for which the Alternating Series Test fails

Determine the convergence or divergence of the following series.

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{n+1}{n}=\frac{2}{1}-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\frac{6}{5}-\ldots$$

Alternating Series Remainder

If a convergent alternating series satisfies the conditions $a_{n+1} \leq a_n$, then the absolute value of the remainder R_n involved in approximating the sum S by S_N is less than or equal to the first neglected term. That is,

$$|S - S_N| = |R_N| \leq a_{N+1}$$

Example: Approximate the sum of the following series by its first six terms.

$$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{n!}$$

Practice

Determine the convergence or divergence of the series.

1. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$$

2. $$\sum_{n=1}^{\infty} \frac{(-1)^{n}}{\ln(n+1)}$$

3. $$\sum_{n=1}^{\infty} \frac{(-1)^{n} n^2}{n^2+1}$$

4. $$\sum_{n=1}^{\infty} \frac{1}{n} \cdot \cos(n\pi)$$

5. $$\sum_{n=0}^{\infty} \frac{(-1)^{n}}{n!}$$

6. $$\sum_{n=1}^{\infty} \frac{1}{n} \cdot \sin\left(\frac{(2n-1)\pi}{2}\right)$$

7. Approximate the sum of the series using the first six terms.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2^n}$$