How do we find power series representations of functions?

Quick Check

Determine whether the following series converges. If it does, find the sum.

$$\sum_{n=0}^{\infty} 6\left(\frac{4}{5}\right)^n$$

Power series representation

Recall Geometric Series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad |r| < 1$$

Consider $f(x) = \dfrac{1}{1-x}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Of course, this series represents $f(x)$ only on the interval $(-1, 1)$. To represent another interval, you must develop another series.

Practice

1️⃣ Find a power series for $f(x) = \dfrac{4}{x+2}$, centered at 0. What is the interval of convergence.

2️⃣ Find a geometric power series centered at 1 for $f(x) = \dfrac{1}{x}$. Then, find the interval of convergence.

3️⃣ Use partial fractions to break the function, then represent it by adding two geometric power series. The interval of convergence is the intersection of individual intervals.

$$f(x) = \frac{3x-1}{x^2-1} \qquad\qquad c = 0$$

Finding a Power Series by Integration

4 Find a power series for $f(x) = \ln x$, centered at 1.

5 Find a power series for $g(x) = \arctan x$, centered at 0.

More Practice.

6 Find a power series for $f(x) = \dfrac{1}{2 - x}$, $c = 5$. Find the interval of convergence.

7 Find the power series for $f(x) = \dfrac{3x}{x^2 + x - 2} \quad c = 0.$