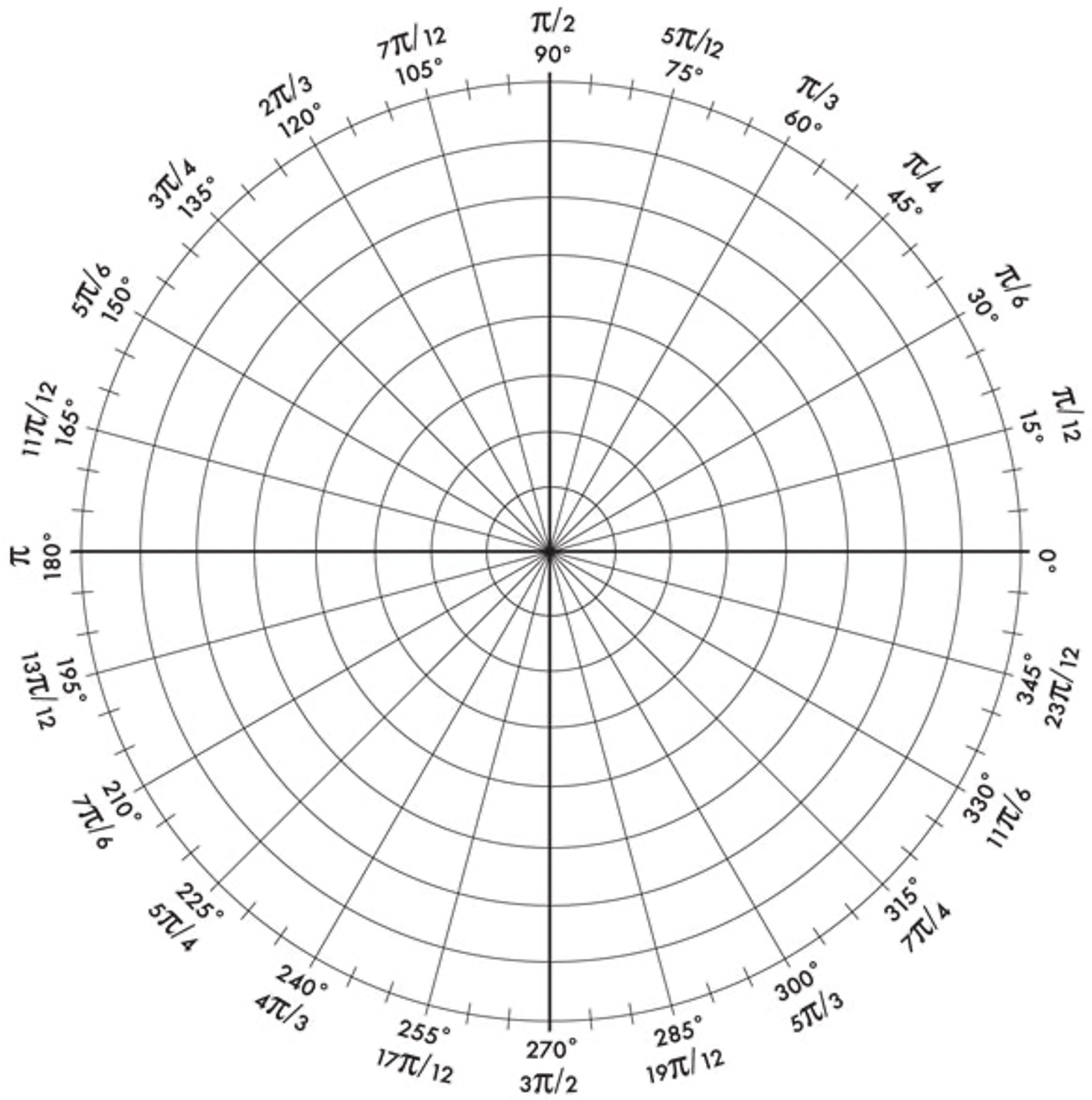
How do we find tangent lines to polar graphs?

Quick Check

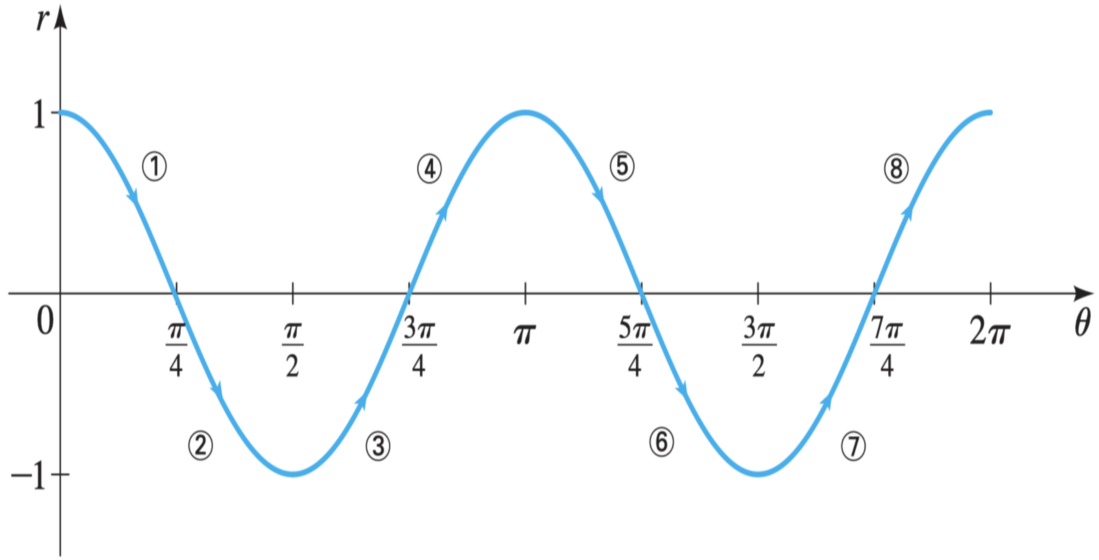
Sketch on Polar Graph paper.

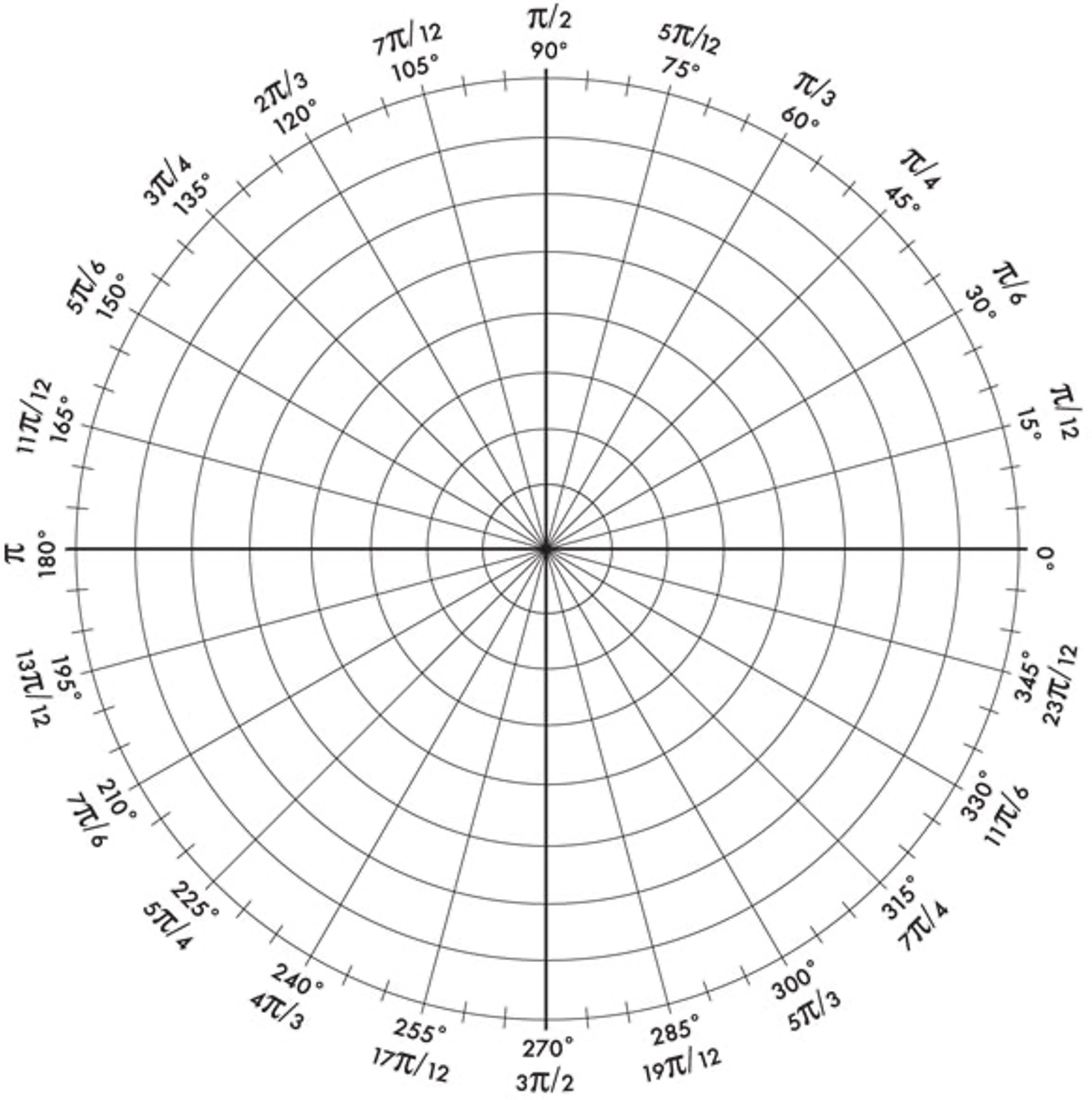
1. $r = 3$
2. $\theta = \pi/3$
3. $r = 2\sin\theta$
4. $r = 3(1 + \cos\theta)$



Sketching using *sine* or *cosine* curve

Sketch $r = \cos 2\theta$





Slope in Polar Form

To find the slope of a tangent line to a polar graph, consider a differentiable function given by $r = f(\theta)$. Use the parametric equations

$$x = r\cos\theta = f(\theta)\cos\theta \quad \text{and} \quad y = r\sin\theta = f(\theta)\sin\theta$$

then use the parametric form of the derivative.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Example

Find the horizontal and vertical tangent lines to $r = sin\theta$ on $0 \leq \theta \leq \pi$.

Note:

1. Horizontal tangents are solutions to $\frac{dy}{d\theta} = 0$

2. Vertical tangents are solutions to $\frac{dx}{d\theta} = 0$, as long as $\frac{dy}{d\theta} \neq 0$.

Practice

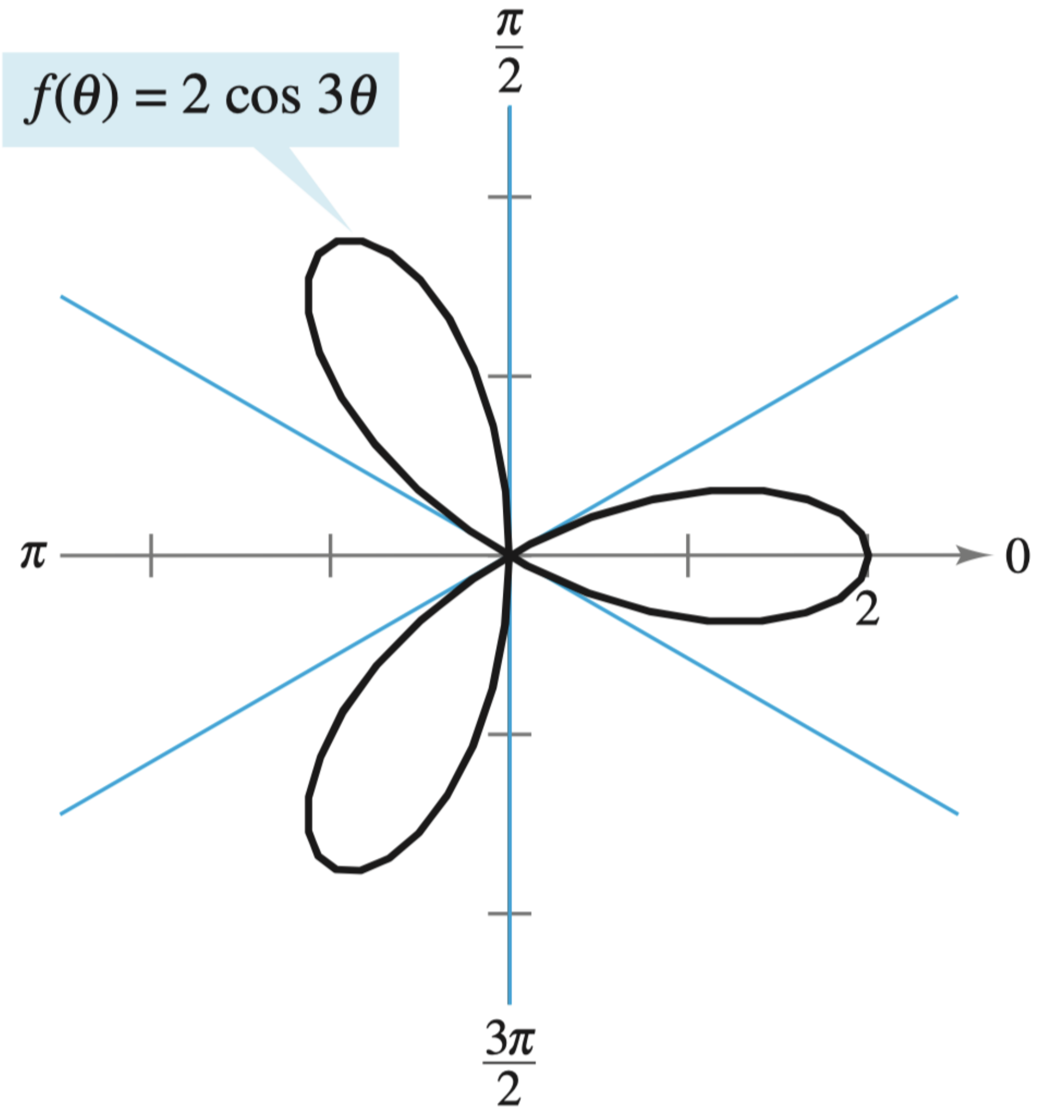
Find the horizontal and vertical tangents to the graph of $r = 2(1 - cos\theta)$.

graph and check your answer

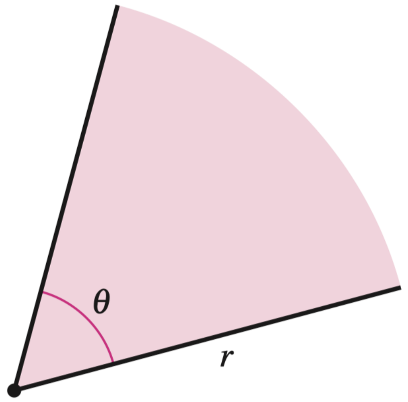
Tangent Lines at the Pole

If $f(a) = 0$ and $f'(a) \neq 0$, then the line $\theta = a$ is tangent at the pole to the graph of $r = f(\theta)$

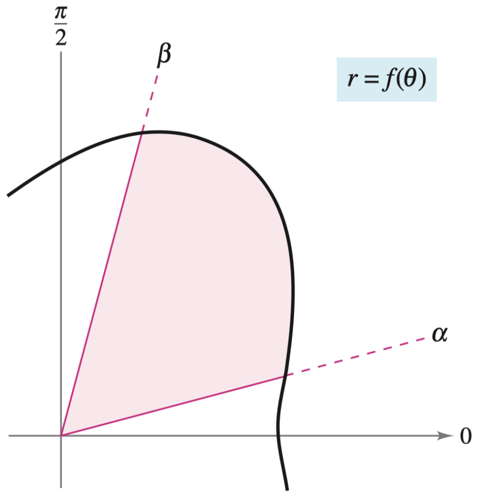
Algebraically find the equations of the tangents at the pole for $r = 2\cos 2\theta$.

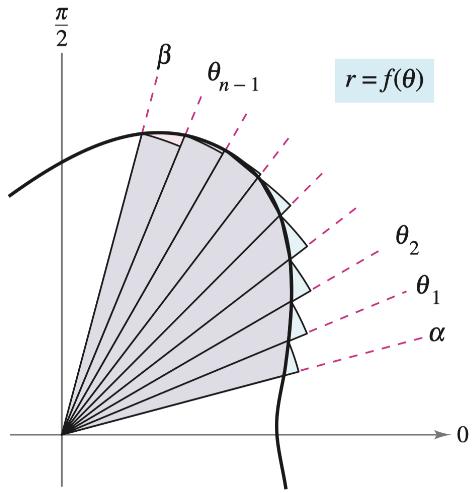


Area of a Polar Region



The area of a sector of a circle is $A = \frac{1}{2}\theta r^2$.





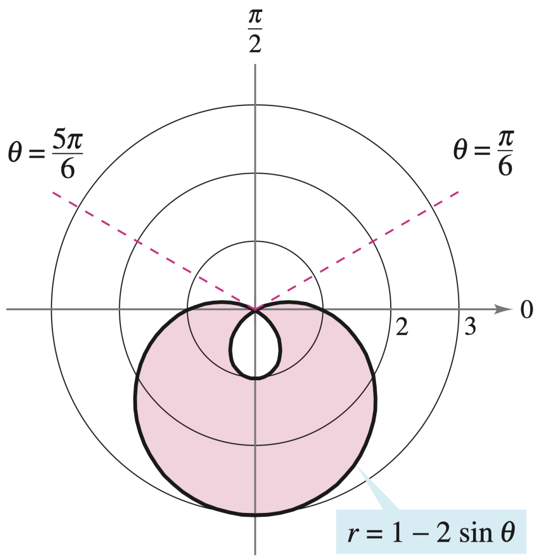
If f is continuous and non-negative on $[\alpha, \beta]$ and $0 \leq \beta - \alpha \leq 2\pi$,

$$Area = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

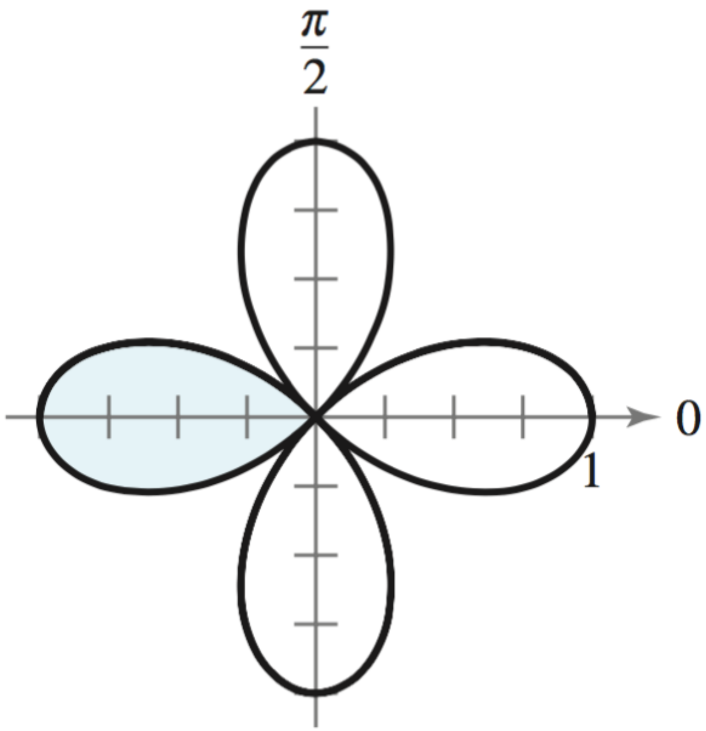
Example

1. Find the area of one petal of the rose curve given by $r = 3\cos 3\theta$. *Start by sketching the region first.*

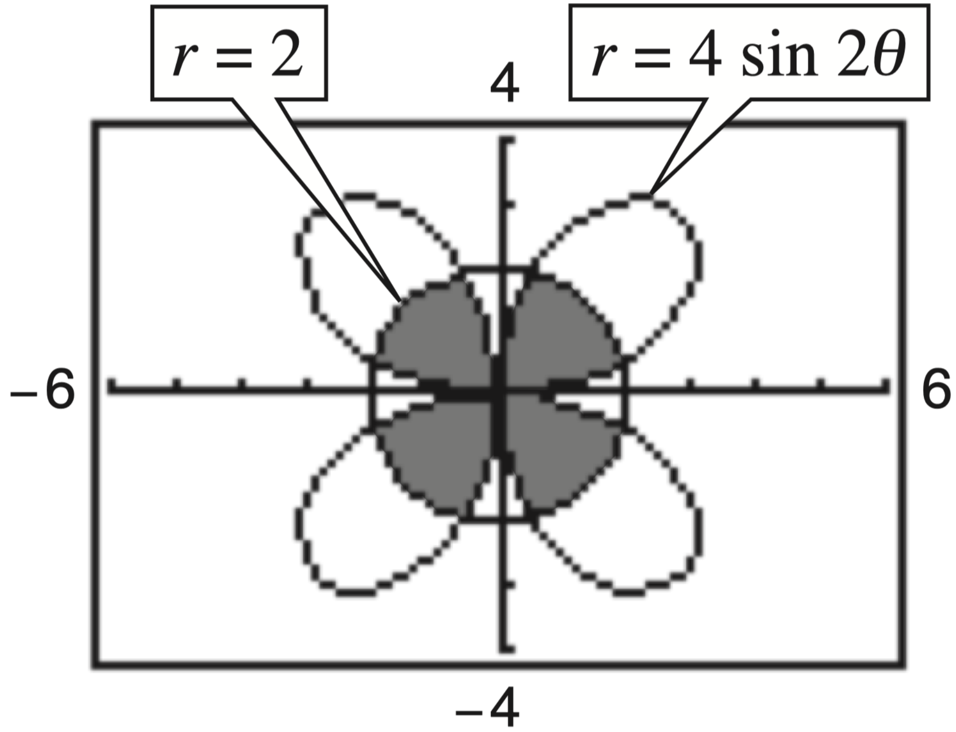
2. Find the area of the region lying between the inner and outer loops of the limacon $r = 1 - 2\sin\theta$.



Setup the integral that represents the area of the region



1 $r = \cos 2\theta$



2 Common interior of $r = 4\sin 2\theta$ and $r = 2$