Lesson 10: How can we compare scores with different units?


The story:

The women's heptathlon in the Olympics consists of seven track and field events: the 200-m and 800-m runs, 100-m high hurdles, shot put, javelin, high jump, and long jump. To determine who should get the gold medal, somehow the performances in all seven events have to be combined into one score. How can performances in such different events be compared? They don't even have the same units; the races are recorded in minutes and seconds and the throwing and jumping events in meters. In the 2004 Olympics, Austra Skujyté of Lithuania put the shot 16.4 meters, about 3 meters farther than the average of all contestants. Carolina Klüft won the long jump with a 6.78-m jump, about a meter better than the average. Which performance deserves more points? Even though both events are measured in meters, it's not clear how to compare them. The solution to the problem of how to compare scores turns out to be a useful method for comparing all sorts of values whether they have the same units or not.

| Long Jump Stem | Long Jump Leaf | Shot Put Stem | Shot Put Leaf |
|---|---|---|---|
| 67 | 8 | 16 | 4 |
| 66 | | 15 | |
| 65 | 1 | 15 | |
| 64 | 2 | 14 | 56778 |
| 63 | 0566 | 14 | 24 |
| 62 | 11235 | 13 | 5789 |
| 61 | 0569 | 13 | 012234 |
| 60 | 2223 | 12 | 55 |
| 59 | 0278 | 12 | 0144 |
| 58 | 4 | 11 | 59 |
| 57 | 0 | 11 | 23 |

Stem-and-leaf displays for both the long jump and the shot put in the 2004 Olympic Heptathlon. Carolina Klüft (green scores) won the long jump, and Austra Skujyté (red scores) won the shot put. Which heptathlete did better for both events combined?

| | Event: Long Jump | Event: Shot Put |
|---|---|---|
| Mean (all contestants) | 6.16 m | 13.29 m |
| SD | 0.23 m | 1.24 m |
| $n$ | 26 | 28 |
| Klüft | 6.78 m | 14.77 m |
| Skujyté | 6.30 m | 16.40 m |

## Standardizing with *z*-Scores

$$z = \frac{y - \bar{y}}{s}$$

These values are called standardized values,and are commonly denoted with the letter **z**. Usually, we just call them **z**-scores.

Standardized values have no units. z-scores measure the distance of each data value from the mean in standard deviations.

A z-score of 2 tells us that a data value is 2 standard deviations above the mean. It doesn't matter whether the original variable was measured in inches, dollars, or seconds. Data values below the mean have negative z-scores, so a z-score of -1.6 means that the data value was 1.6 standard deviations below the mean.

The farther a data value is from the mean, the more unusual it is.

## Standardizing skiing times

The men's combined skiing event in the winter Olympics consists of two races: a downhill and a slalom. Times for the two events are added together, and the skier with the lowest total time wins. In the 2006 Winter Olympics, the mean slalom time was 94.2714 seconds with a standard deviation of 5.2844 seconds. The mean downhill time was 101.807 seconds with a standard deviation of 1.8356 seconds. Ted Ligety of the United States, who won the gold medal with a combined time of 189.35 seconds, skied the slalom in 87.93 seconds and the downhill in 101.42 seconds.

Question: On which race did he do better compared with the competition?

Who should win?

| | | Event: Long Jump | Event: Shot Put |
|---|---|---|---|
| | Mean | 6.16 m | 13.29 m |
| | SD | 0.23 m | 1.24 m |
| Klüft | Performance | 6.78 m | 14.77 m |
| | $z$-score | $\frac{6.78 - 6.16}{0.23} = 2.70$ | $\frac{14.77 - 13.29}{1.24} = 1.19$ |
| | Total $z$-score | $2.70 + 1.19 = \mathbf{3.89}$ | |
| Skujyté | Performance | 6.30 m | 16.40 m |
| | $z$-score | $\frac{6.30 - 6.16}{0.23} = 0.61$ | $\frac{16.40 - 13.29}{1.24} = 2.51$ |
| | Total $z$-score | $0.61 + 2.51 = \mathbf{3.12}$ | |

Just Checking

Your Statistics teacher has announced that the lower of your two tests will be dropped. You got a 90 on test 1 and an 80 on test 2. You're all set to drop the 80 until she announces that she grades "on a curve." She standardized the scores in order to decide which is the lower one. If the mean on the first test was 88 with a standard deviation of 4 and the mean on the second was 75 with a standard deviation of 5,

**a)** Which one will be dropped?
**b)** Does this seem "fair"?