Lesson 25: What are basic probability rules?

John Venn, FRS,[2][3] FSA,[4] (4 August 1834 – 4 April 1923) was an English logician and philosopher noted for introducing the Venn diagram, used in the fields of set theory, probability, logic, statistics, competition math, and computer science. In 1866, Venn published The Logic of Chance, a ground-breaking book which espoused the frequency theory of probability, offering that probability should be determined by how often something is forecast to occur as opposed to "educated" assumptions. Venn then further developed George Boole's theories in the 1881 work Symbolic Logic, where he highlighted what would become known as Venn diagrams.
-wikipedia.org

First 3 Rules

1. Make a picture

2. Make a picture

3. Make a picture

Venn Diagrams help us think about probabilities.

Do you want to lose money??

What is one feature
of games of chance?
...like those
involving number cubes
and spinners.

It is easy to find probabilities of events that are equally likely

$$P(\mathbf{A}) = \frac{\#\text{ outcomes in } \mathbf{A}}{\#\text{ of possible outcomes}}$$

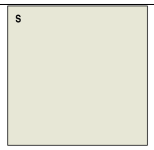
For example, the probability of drawing a face card in a deck is

What does it mean?

For any event **A**, $0 \le P(\mathbf{A}) \le 1$

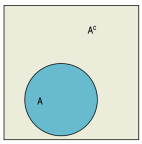
P(A) = 0 ??

P(A) = 1 ??

$P(\mathbf{S}) = 1$ (**S** represents the set of all possible outcomes.)	
Suppose a six sided die is rolled, what is the sample space (**S**).	 The sample space **S**.

Complement Rule

The set of outcomes that are *not* in the event **A** is called the complement of **A**, denoted $\mathbf{A}^c$.

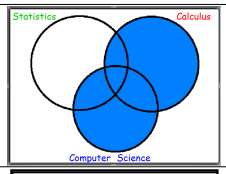
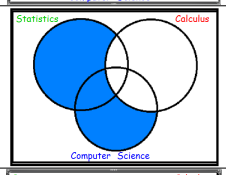
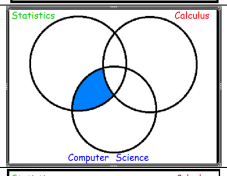
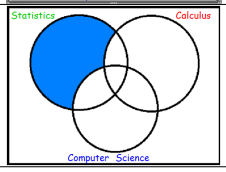


The set **A** and its complement.

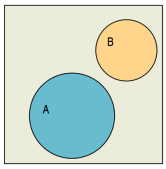
The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(\mathbf{A}) = 1 - P(\mathbf{A}^c)$
Suppose a six-sided die is rolled. The event that the die would land on an even number would be E = {2, 4, 6} What would the event be that is the die **NOT** landing on an even number?
E^c = {1, 3, 5}

Practice with Venn Diagrams

Some students are enrolled in Statistics, some in Calculus, and some in Computer Science.
Indicate what relationships the shaded regions represent. (use complement, intersection, and union)

	Calculus or Computer Science
	(Statistics or Computer Science) and not Calculus
	Statistics and Computer Science and not Calculus
	Statistics and not (Computer Science or Calculus)

Disjoint or mutually exclusive



Two disjoint sets, **A** and **B**.

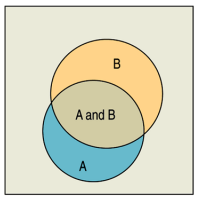
Events that have no outcomes in common (and, thus, cannot occur together)

provided that **A** and **B** are disjoint.

$$P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$$

Independent Events

the outcome of one event doesn't influence the outcome of another



Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$$

Class Examples:

1. Suppose that 40% of cars in your area are manufactured in the United States, 30% in Japan, 10% in Germany, and 20% in other countries. If cars are selected at random, find the probability that:

- A car is not U.S.-made.

- It is made in Japan or Germany.

- You see two in a row from Japan.

- None of three cars came from Germany.

- At least one of three cars is U.S.-made.

- The first Japanese car is the fourth one you choose.

For calculating probabilities, remember three simple ideas, based on three words:

- **"not"** means subtract from 1. If 30% of people have blue eyes, then the other 70% of people do not.
- **"or"** means add. The probability that your eyes are green or brown is the sum of the probability of having green eyes and the probability of having brown eyes. That's true, of course, only if you can't have both green and brown eyes.
- **"and"** means multiply. The probability that two students sitting near each other in class both have blue eyes is the product of the probabilities that each has blue eyes. That's true, of course, only if their eye colors are independent. If they were twins, all bets are off. It's enough right now to point that out.

Five multiple choice questions, each with four possible answers, appear on your history exam. What is the probability that if you just guess, you

a. get none of the questions correct?

b. get all of the questions correct?

c. get at least one of the questions wrong?

d. get your first incorrect answer on the fourth question?

The Masterfoods company manufactures bags of Peanut Butter M&M's. They report that they make 10% each brown and red candies, and 20% each yellow, blue, and orange candies. The rest of the candies are green.

a. If you pick a Peanut Butter M&M at random, what is the probability that
   i. it is green?

   ii. it is a primary color (red, yellow, or blue)?

   iii. it is not orange?

b. If you pick four M&M's in a row, what is the probability that
   i. they are all blue?

   ii. none are green?

   iii. at least one is red?

   iv. the fourth one is the first one that is brown?