Lesson 27: Can disjoint events be independent?

Are "face card" and "king" independent? Mutually Exclusive?

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

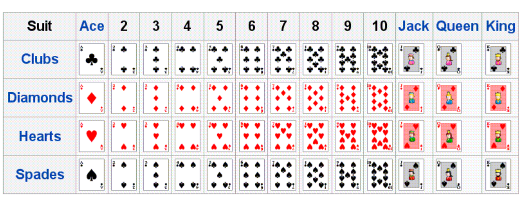
Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

I draw a card from a deck. You have to bet on whether or not it's red. You have a 50-50 chance of being right. Now suppose that before you bet, I peek at the card and tell you it's a spade. Does that help you place your bet? Of course! Note that the events "red" and "spade" are disjoint – there are no red spades. Because of that, knowing the card is a spade is a dead giveaway; it tells you for sure that the card can't be red. The probability the card is red has changed from 1/2 to 0. These two events are NOT independent.

I draw a card from a deck. You have to bet on whether or not it's red.

OK, let's play again. This time I peek at the card and tell you it's an ace. Does that help you place your bet? Before you knew this, the probability the card is red was 26/52 = 1/2. Knowing it's an ace, the probability it's red is 2/4 = 1/2. No help whatsoever – the probability has not changed. These two events ARE independent (and not disjoint). P(red | ace) = P(red) — that's the very definition of independence:

the occurrence of "ace" has no effect on the probability of "red".



One card is drawn. What is the probability it is an ace or red? (General Addition Rule)
Answer:
Think – The events "ace" and "red" are not disjoint. There are 2 red aces in the deck. The General Addition Rule can be used.

Show – $P(\text{ace or red}) = P(\text{ace}) + P(\text{red}) - P(\text{ace and red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$

Tell – The probability that a randomly selected card is an ace or red is $\frac{28}{52}$.

Two cards are drawn without replacement. What is the probability they are both aces? (General Multiplication Rule) Extend to the probability of getting 5 hearts in a row.
Answer:
Think – Two cards are drawn without replacement. The two draws are not independent. The General Multiplication Rule can be used.

Show – $P(\text{both cards are aces}) = P(\text{ace})P(\text{ace}|\text{first draw was ace}) = \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \frac{12}{2652}$

Tell – The probability that two randomly selected cards are both aces is $\frac{12}{2652}$.

$P(5 \text{ hearts}) = \left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)\left(\frac{10}{49}\right)\left(\frac{9}{48}\right) \approx 0.0005$

I draw one card and look at it. I tell you it is red. What is the probability it is a heart? And what is the probability it is red, given that it is a heart? (Conditional probability)
Answer:
Think – The card is selected at random.

Show – $P(\text{heart}|\text{red}) = \dfrac{P(\text{heart and red})}{P(\text{red})} = \dfrac{\frac{13}{52}}{\frac{26}{52}} = \dfrac{1}{2}$ (half of the red cards are hearts)

$P(\text{red}|\text{heart}) = \dfrac{P(\text{red and heart})}{P(\text{heart})} = \dfrac{\frac{13}{52}}{\frac{13}{52}} = 1$ (all of the hearts are red)

Tell – If a randomly chosen card is a red, the probability that it is a heart is 0.5. If a randomly chosen card is a heart, it is certain to be red.

P(heart | Red) = P(heart)

- Are "red card" and "spade" independent? Mutually exclusive?
  Answer: "Red card" and "spade" are not independent events. There's a 25% chance a card is a spade, but if you know that you have a red card, you are certain that you don't have a spade.
  The events are mutually exclusive (disjoint), since there are no red cards that are also spades.

- Are "red card" and "ace" independent? Mutually exclusive?
  Answer: "Red card" and "ace" are independent events. The probability that a card is red is 0.5. The probability that a card chosen from the four aces is red is also 0.5.
  $P(\text{Red}) = P(\text{Red} \mid \text{Ace})$
  The events are not mutually exclusive (not disjoint), since there are two red aces.

- Are "face card" and "king" independent? Mutually exclusive?
  Answer: "Face card" and "king" are not independent. The probability of drawing a face card is $\frac{12}{52}$. The probability that a face card is drawn from the four kings is 1.
  $P(\text{face}) \neq P(\text{face} \mid \text{King})$
  The events are not mutually exclusive (not disjoint), since all four kings are face cards.

Are events that are not disjoint necessarily independent?

Events that are not disjoint may or may not be independent. Baseball games are sometimes played in the rain, so "baseball" and "rain" are not mutually exclusive. But they're not independent either because games are less likely to be played when it rains.

Colorblindness occurs more often in men than in women. A woman can be colorblind, so the events are not disjoint. But the probability is much lower, so gender and colorblindness are not independent either

Can disjoint events be independent?

If the events are disjoint then they cannot both happen at the same time. That means that as soon as one of them happens the other cannot – the probability of the other changes to 0. When the occurrence of one event changes the probability of the other, the events are not independent.

Define independence using conditional probability

2-way table: males or females wearing jeans or not

	Jeans	Other	Total
M			
F			
Total			

- What is the probability that a male wears jeans?
  Answer:

- What is the probability that someone wearing jeans is a male?
  Answer:

- Are being male and wearing jeans disjoint?

- Are sex and attire independent? Have students explain in context what they need to know to ascertain independence. There are many ways to check for independence; write several such statements down, in words and formula.

When we want the probability of an event from a *conditional* distribution,we write **P(B|A)** and pronounce it "the probability of **B** *given* **A.**"A probability that takes into account a given *condition* such as this is called a **conditional probability**

To find the probability of the event **B** *given* the event **A,** we restrict our attention to the outcomes in **A.** We then find in what fraction of *those* outcomes **B** also occurred. Formally, we write,

$$P(\mathbf{B} \mid \mathbf{A}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{A})}$$

Finding a conditional probability

Our survey found that 56% of college students live on campus, 62% have a campus meal program, and 42% do both.

Question: While dining in a campus facility open only to students with meal plans, you meet someone interesting. What is the probability that your new acquaintance lives on campus?

Let $\mathbf{L}$ = {student lives on campus} and $\mathbf{M}$ = {student has a campus meal plan}.

$$P(\text{student lives on campus given that the student has a meal plan}) = P(\mathbf{L} \mid \mathbf{M}) = \frac{P(\mathbf{L} \cap \mathbf{M})}{P(\mathbf{M})} = \frac{0.42}{0.62} \approx 0.677$$

There's a probability of about 0.677 that a student with a meal plan lives on campus

Events **A** and **B** are **independent** whenever

$$P(\mathbf{B} \mid \mathbf{A}) = P(\mathbf{B})$$

Recap: Our survey told us that 56% of college students live on campus, 62% have a campus meal program, and 42% do both.

Question: Are living on campus and having a meal plan independent? Are they disjoint?

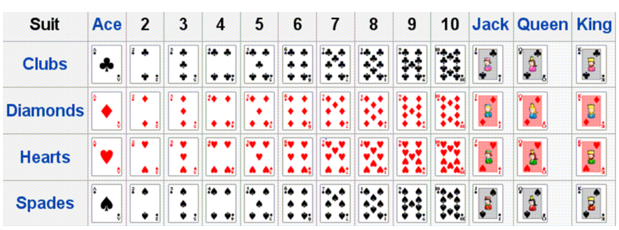
Let $\mathbf{L}$ = {student lives on campus} and $\mathbf{M}$ = {student has a campus meal plan}. If these events are independent, then knowing that a student lives on campus doesn't affect the probability that he or she has a meal plan. I'll check to see if $P(\mathbf{M} \mid \mathbf{L}) = P(\mathbf{M})$:

$$P(\mathbf{M} \mid \mathbf{L}) = \frac{P(\mathbf{L} \cap \mathbf{M})}{P(\mathbf{L})} = \frac{0.42}{0.56} = 0.75, \quad \text{but } P(\mathbf{M}) = 0.62.$$

Because $0.75 \neq 0.62$, the events are not independent; students who live on campus are more likely to have meal plans. Living on campus and having a meal plan are not disjoint either; in fact, 42% of college students do both.

Multiplication Rule can be written without the requirement for independence.

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B} \mid \mathbf{A})$$



Cards. You draw a card at random from a standard deck of 52 cards. Find each of the following conditional probabilities:

a) The card is a heart, given that it is red.
b) The card is red, given that it is a heart.
c) The card is an ace, given that it is red.
d) The card is a queen, given that it is a face card.

Health. The probabilities that an adult American man has high blood pressure and/or high cholesterol are shown in the table.

Cholesterol	Blood Pressure: High	Blood Pressure: OK
High	0.11	0.21
OK	0.16	0.52

What's the probability that
a) a man has both conditions?
b) a man has high blood pressure?
c) a man with high blood pressure has high cholesterol?
d) a man has high blood pressure if it's known that he has high cholesterol?

Birth order. A survey of students in a large Introductory Statistics class asked about their birth order (1 = oldest or only child) and which college of the university they were enrolled in. Here are the results:

College	Birth Order: 1 or only	Birth Order: 2 or more	Total
Arts & Sciences	34	23	57
Agriculture	52	41	93
Human Ecology	15	28	43
Other	12	18	30
Total	113	110	223

a) If we select a student at random, what's the probability the person is an Arts and Sciences student who is a second child (or more)?
b) Among the Arts and Sciences students, what's the probability a student was a second child (or more)?
c) Among second children (or more), what's the probability the student is enrolled in Arts and Sciences?
d) What's the probability that a first or only child is enrolled in the Agriculture College?
e) What is the probability that an Agriculture student is a first or only child?

Death penalty. The table shows the political affiliations of American voters and their positions on the death penalty.

Party	Death Penalty: Favor	Death Penalty: Oppose
Republican	0.26	0.04
Democrat	0.12	0.24
Other	0.24	0.10

a) What's the probability that
   i) a randomly chosen voter favors the death penalty?
   ii) a Republican favors the death penalty?
   iii) a voter who favors the death penalty is a Democrat?

b) A candidate thinks she has a good chance of gaining the votes of anyone who is a Republican or in favor of the death penalty. What portion of the voters is that?